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Физика: Механика

Учебное пособие
для иностранных студентов медицинских вузов
(на английском языке)

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В учебном пособии представлены современные сведения о законах и явлениях, необходимых для понимания физических методов исследования свойств и характеристик биологических объектов и дальнейшего использования полученных результатов в медицине. Пособие содержит главы по основным темам раздела «Механика» курса общей физики, а также список контрольных вопросов и тестовые задания. Предложенная структура пособия помогает выделить главные аспекты изучаемых физических процессов, организовать и конкретизировать учебный процесс.

Данное пособие подготовлено в соответствии с учебной программой по дисциплине «Физика, математика» для иностранных студентов, обучающихся по специальности «Лечебное дело» (билингвальная форма обучения).

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ОГЛАВЛЕНИЕ

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Physics: Mechanics

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In this tutorial edition, we reflect on the current informaion about laws and phenomena, which are necessary for understanding the physical methods of research the properties and characteristics of biological objects for further use of the obtained results in medical practice. There main topics of section “Mechanics” of general physics course are presented, as well as control questions and test tasks. The proposed structure of this tutorial edition helps to highlight the main aspects of studying physical processes, and organize and reinforce the educational process.

This edition was prepared on the discipline "Physics, mathematics" in accordance with the training program of foreign students enrolled in the "General Medicine" specialty (bilingual education system).

Reviewer: O.N. Sulakshina PhD, Professor of Division of physics with course of higher mathematics, SSMU (Tomsk, Russian Federation)

CONTENTS

INTRODUCTION

The task of physics is to create the most complete description of the physical properties of the world. In the study of physics as a science, it is very important to bear in mind the model character of its constructions. Meeting in daily life and practice of different physical objects, events, situations and relationships between them and man creates a model, which consists of images of these objects, events, situations, and relationships between them, and the rules of operating with them. In the real world of physical phenomena, the relationship between objects is so diverse that it is impossible to cover them all, not only practically, but also in a theoretical sense. Therefore, the creation of models takes into account only the essential information and conditions for the range of phenomena and properties of the connection. It is necessary to ensure that every element of the studied model has a clearly defined content and has articulated a relationship with an element of the real physical world.

Physics in the modern system of Sciences studies the most common and simplest forms of matter in motion (mechanical, thermal, electromagnetic, etc.) and their mutual transformation. The laws of physics, such as the law of conservation of energy, the laws of electrodynamics, and quantum mechanics, are the basis of chemical and biological laws. Clear boundaries cannot be set between physics and other sciences of nature. For example, application of theoretical and experimental physics methods has enabled the determination of the structure and properties of the primary particle involved in a chemical process: the atoms, molecules, and free radicals. These methods allow, in many cases, the understanding of the details of chemical reactions, to find out the mechanism and kinetics of chemical reactions, and to establish the nature of chemical bonds.

CHAPTER 1. KINEMATICS

1.1. Vectors and scalars

Many physical values are characterized by a single number. These include, for example, temperature, expressed by the number of degrees in a certain scale; mass - the number of grams, etc. These values are called scalars.

There are characteristics such as velocity, given as the number and direction. These characteristics are called vectors.

The vector is a directed line segment, the length of which is represented by a vector physical quantity, and the arrow indicates its direction. Sometimes vectors are denoted in bold letters, for example, **A**, and their absolute value - or the same bold letters concluded between the vertical bars: |**A**| or the same letter, but a light font.

Since the vectors are characterized by the direction and magnitude, work with vector values need special rules.

1. Addition of vectors. Addition of vectors $\vec{a}$ and $\vec{b}$ is carried out either by the rule of the triangle (Figure 1.1) or by the rule of parallelogram (Figure 1.2).

Consider two vectors $\vec{a}$ and $\vec{b}$ (Figure 1.1). Vector $\vec{a}$ transference the parallel to itself so that it turned out to be the beginning of combined with the end of the vector $\vec{b}$. Then the vector $\vec{c}$, conducted by the beginning of the vector $\vec{b}$ in the end of the vector $\vec{a}$, will be a resultant vector $\vec{c} = \vec{a} + \vec{b}$

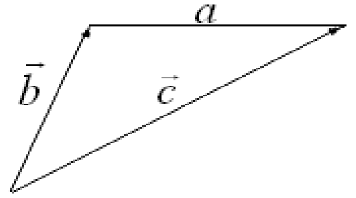

Figure 1.1

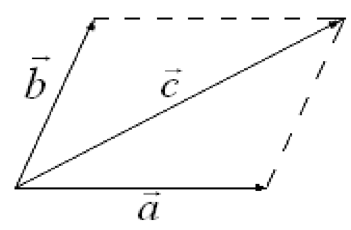

Figure 1.2

It is possible to carry out the construction of another method shown in Figure 1.2. We transfer the vector of $\vec{a}$ and $\vec{b}$, so that the beginning of the two vectors seemed combined. Then constructed on the vectors $\vec{a}$ and $\vec{b}$ is a parallelogram. The diagonal parallelogram coincides with the vector $\vec{c}$, obtained by the method shown in Figure 1.1, i.e. the two discussed ways to give the same result.

2. Very often, implementation of specific numerical calculations is much easier if you work with vectors in coordinate form. In this case, calculations are arithmetic. It is therefore important to know how to record all transactions and expression vector in a coordinate manner. First of all, it is necessary to know how to do in Cartesian coordinates. In this case, any vector can be projected on the coordinate axes and the projections of this vector are as follows:

$$A_x = x_2 - x_1,\ A_y = y_2 - y_1,\ A_z = z_2 - z_1$$

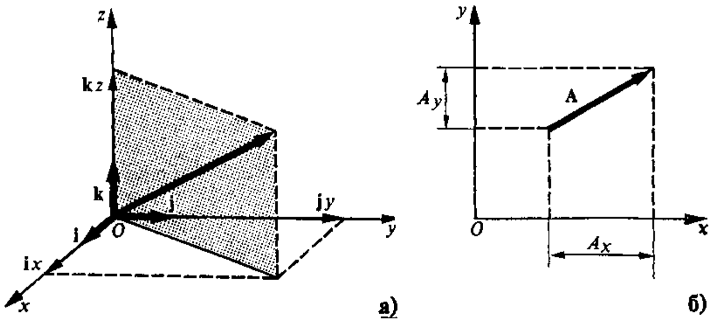

Figure 1.3

Figure 1.3 shows a projection of an arbitrary vector in a spatial cartesian coordinate system (*a*) and an arbitrary vector A on the same system plane (*б*).

The figure 1.3 it is seen that the modulus of vector may be expressed as follows:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3. Convenient vector quantities recording is recording by their unit vectors - a vector whose absolute value is equal to one and the direction corresponds to the vector itself. Therefore, any vector can be expressed as

$$\vec{A} = \vec{e}_a A, \qquad (1)$$

where A - the module vector, $\vec{e}_a$ – a single vector or unit vector A, directed in the same way as the vector $\vec{A}$. Multiplying both sides of equation (1) by a scalar, equal to $1/A$, we arrive at the relation

$$\vec{e}_a = \frac{\vec{A}}{A}$$

This relationship implies that the unit is a dimensionless quantity.

1.2. Material point. Reference system

Mechanical motion is the change of the position of a body or its parts relative to other bodies over time.

The body, which we have seen with respect to the position of other bodies in space, called *the body of reference*.

Section mechanics deals with the motion of material bodies without consideration of the causes of this movement, known as *kinematics*.

The simplest mechanical motion is the motion of a material point. *Material point* is a body, the size of which is neglected in the analysis of body movements. The model is applicable when body size is much larger than the size of the region of space in which there is a movement.

If the body cannot be considered as a material point, it is presented as a set of (system) of material points. For example, a solid - a system rigidly interconnected material points; an elastic body - the system of material points, capable of relatively small movements; gas - system of non-related material points.

To describe the mechanical movement, it is necessary to indicate how one body moves relative to any other material bodies. Therefore, first, set the reference system.

Frame of reference refers to a coordinate system associated with the body of reference, and the clock, which are in the system of coordinates and the time.

Material point position in the coordinate system at time t is determined by the coordinates x, y, z or the radius vector $\vec{r}$. *The radius vector ($\vec{r}$) is the vector drawn from the origin to the point in space, which at a given time is the body* (see Figure 1.4). Over time, the position of the material point changes - it moves, i.e. the radius vector $\vec{r}$ and three scalar values x, y, z are functions of time: $\vec{r} = \vec{r}(t)$ or

$$x = x(t),$$
$$y = y(t),$$
$$z = z(t)$$

The set of successive positions occupied by the point P in the process of movement in space forms a curve called *the trajectory of a moving point*.

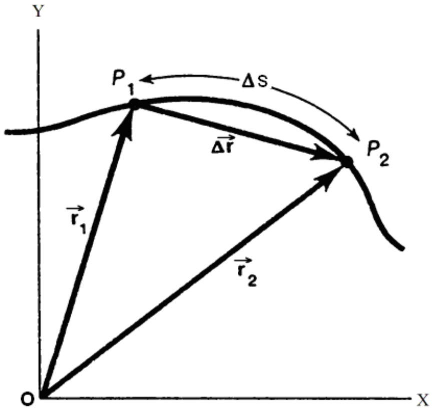

Figure 1.4

The trajectory of the particle in the plane shown in Figure 1.4. At time t_1 the particle is at the point P_1 (its position is given by the radius vector r_1), and at time t_2 at the point P_2 (its position is given by the radius vector r_2). Vector move within the time interval Δt is $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$.

Trajectory - a curve that describes the end of the radius vector of the motion of a material point. If the motion of a material point P changes only the length of the radius vector, the point P is moving in a straight line. Such movement is called *rectilinear*. When changing only the direction of the radius vector of the motion trajectory of the point P, the movement is circular. If the radius vector of point P varies both in direction and in magnitude, the trajectory of the motion will be submitted to a curve (*curvilinear motion)*. The length of the trajectories traveled by the material point in time Δt, called the length of the path ΔS.

The directed line segment connecting the start and end point of the movement is called *the movement*. The vector $\Delta\vec{r} = \vec{r} - \vec{r}_0$ in Figure 1.4 is a movement of a material point P. If we consider an infinitesimal displacement of a material point along a path, then $\Delta S = |\Delta\vec{r}|$.

1.3. Kinematics of material point

Of course, we know that the relation to move $\Delta\vec{r}$ to the time Δt is the average movement velocity of movement.

Instantaneous velocity of a moving point corresponds to the average velocity of movement when the movement becomes very small. Mathematically, this definition can be written as:

$$\bar{u} = \lim_{\Delta t \to 0} \frac{\Delta \bar{r}}{\Delta t}$$

The expression on the right-hand side corresponds to the derivative of the radius vector of a moving material point in time:

$$\vec{u} = \frac{d\vec{r}}{dt}$$

It should be noted that we do not just put $\Delta t \to 0$, as the value of Δr in this case would also be equal to zero, and we would have an indefinite number.

Thus, the ratio $\frac{\Delta r}{\Delta t}$ must be considered as a whole; because we believe $\Delta t \to 0$, Δr also tends to zero, but the ratio of $\frac{\Delta r}{\Delta t}$ approaches a certain value, which is called the *instantaneous velocity*.

Let us note that the velocity is a vector quantity. Instantaneous velocity vector is always directed at a tangent to the trajectory at the relevant point (see Figure 1.5).

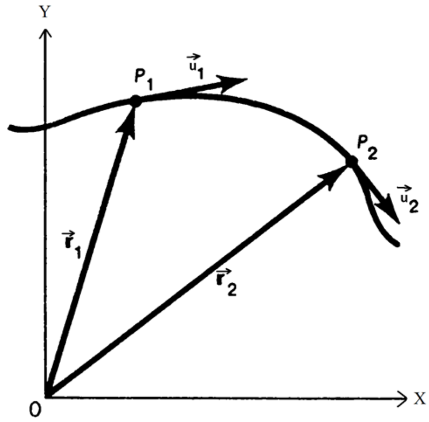

Figure 1.5. Velocity vectors u_1 and u_2, respectively, at the times t_1 and t_2 to the particles in Figure 1.4

Modulus of the velocity vector can be found as follows:

$$u = \frac{ds}{dt}$$

It is possible to solve the inverse problem of kinematics, i.e. set value for the speed you can find the path traversed by a material point in the time interval from t_1 to t_2:

$$s = \int_{t_1}^{t_2} u(t)dt$$

In the case of non-uniform movement, it is necessary to know the law of velocity variation with time. *Average acceleration* - a physical quantity that characterizes the rate of change of velocity with time: $\vec{a} = \dfrac{\vec{u}_2 - \vec{u}_1}{t_2 - t_1}$, where $\vec{u}_1$ and $\vec{u}_2$ – instantaneous velocity at time points t_1 and t_2. As follows from the definition of acceleration – a vector quantity, whose direction is always coincides with the vector change of velocity $\Delta\vec{u} = \vec{u}_2 - \vec{u}_1$. Figure 1.6 shows the vectors of instantaneous velocities at

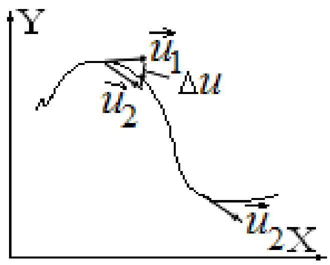

Figure 1.6

different times. To compare these velocities, make a parallel translation of the vector $\vec{u}_2$ at the beginning of the vector $\vec{u}_1$.

Then the vector change of velocity $\Delta\vec{u}$ is directed from the end of the vector $\vec{u}_1$ by the end of the vector $\vec{u}_2$.

Like velocity there is the concept of instantaneous acceleration, which is defined as follows:

$$\vec{a}_{int} = \frac{d\vec{u}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Acceleration of a material point in a given time - is a physical quantity equal to the limit of the ratio change of velocity to the length of time for which the change occurs, tends to zero period. Mathematically, this is expressed as follows:

$$\vec{a} = \frac{d\vec{u}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

We write the velocity vector through the unit vector (see Eq 1) $\vec{u} = \vec{e}_u u$. Since the velocity of motion of a point is a vector quantity, its change can occur both in magnitude and direction, so we find the instantaneous acceleration by taking the derivative of velocity with respect to time (to find the derivative of the work):

$$\vec{a} = \frac{d\vec{u}}{dt} = \frac{d}{dt}(\vec{e}_u u) = \vec{e}_u \frac{du}{dt} + \frac{d\vec{e}_u}{dt} u.$$

Thus, the acceleration can be represented as the sum of two independent members, one of which defines the change in velocity in value, and the other - in direction. Changing the velocity in magnitude characterizes tangential acceleration of a material point, which is directed at a tangent to the path, i.e., it coincides in direction with the velocity and the magnitude of the tangential acceleration is as follows:

$$a_\tau = \frac{du}{dt}.$$

Changing the velocity toward characterized normal acceleration $\vec{a}_n$, which is always perpendicular to the velocity vector directed toward the center and radius of curvature of the path along which the material point moves. The value of the normal acceleration is given by

$$a_n = \frac{u^2}{R},$$

where u – the value of the instantaneous velocity of a particle in a given point of the trajectory, R – the radius of curvature of the trajectory.

Full acceleration of a material point is the vector sum of the tangential and normal acceleration

$$\vec{a} = \vec{a}_\tau + \vec{a}_n$$

Full acceleration module is as follows:

$$|\vec{a}| = \sqrt{a_\tau^2 + a_n^2}$$

1.4. Types of mechanical motion of a material point

1. Uniform rectilinear motion. *Uniform rectilinear motion* is the motion in which the material point for any equal intervals makes the same movement. The vector velocity does not change either in magnitude or direction, the magnitude of movement of the body is a traversed path and instantaneous and average velocity are the same. This movement is described by the equation of motion

$$x = x_0 + ut \text{ or } s = ut$$

2. Uniformly accelerated rectilinear motion. *Uniformly accelerated rectilinear motion* is a motion in which the vector of tangential acceleration is constant, i.e., it does not change either in magnitude or direction ($\vec{a}_\tau = \text{const}$), while the acceleration is zero $\vec{a}_n = 0$. In this case full acceleration is equal to the tangential and equal $a = \frac{du}{dt}$, and velocity at any point in time the particle motion is as follows: $u = u_0 + at$. The path traversed by a material point, a given motion is as follows:

$$s = \int u dt = \int (u_0 + at) dt = u_0 t + \frac{at^2}{2}$$

or in coordinate form:

$$x = x_0 + u_0 t + \frac{at^2}{2}$$

3. Uniform motion in a circle. *Uniform motion in a circle* is a motion in which the tangential acceleration is zero $\vec{a}_\tau = 0$, and the acceleration changes in direction, but does not change the value of $|\vec{a}_n| = \text{const}$, in this case, the trajectory of a material point is a circle. To describe the motion of a material point in a circle if you can with the help of linear characteristics (u, a, s) and using the angular characteristics: angular path φ, the angular velocity $\omega = \frac{d\varphi}{dt}$. Consider the relationship between linear and angular characteristics of the motion of a point. Moving from point 1 to point 2 (see. Figure 1.7), the material point traverses a path S, equal to the length of the arc of a circle of radius R and rotates φ, which is equal to the angular and path traveled by a material point. Angle φ - a central angle therefore $S = \varphi R$.

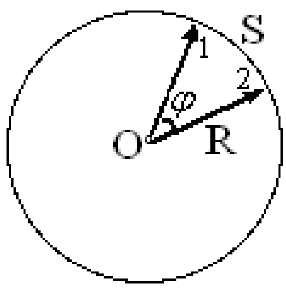

Figure 1.7

Using this connection between the linear and angular traveled distance, we get

$$u = \frac{ds}{dt} = \frac{d(\varphi R)}{dt} = R\frac{d\varphi}{dt} = \omega R$$

Thus the linear velocity u is the angular velocity as follows:

$$u = \omega R$$

Since the uniform circular motion is characterized by a constant normal acceleration in magnitude, then it is expressed through the angular characteristics of the module:

$$|\vec{a}_n| = \frac{u^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R.$$

Thus, the module is normal acceleration $a_n = \omega^2 R$

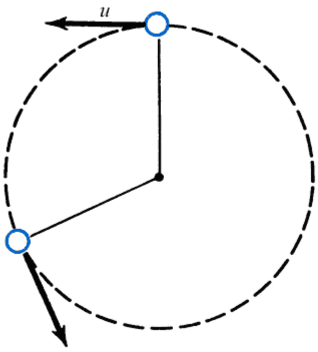

Figure 1.8

Figure 1.8 shows the change in the direction of the instantaneous velocity of a particle moving in a circle. Note that at any given time the instantaneous velocity tangential to the circular path.

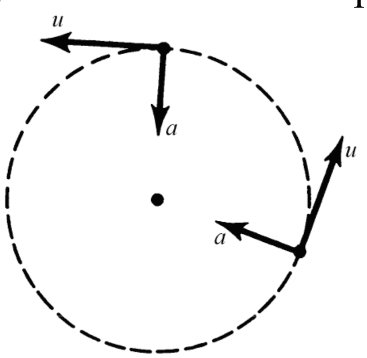

Figure 1.9

Figure 1.9 shows that the uniform rotary motion vector normal acceleration is always perpendicular to the velocity vector u.

4. Accelerated circular motion. With a uniform rotational motion acceleration of the material point has both tangential (a_τ) and normal (a_n) components. In this case, changes in the magnitude of linear velocity, and hence the angular velocity (see Figure 1.10 and the equation $u = \omega R$).

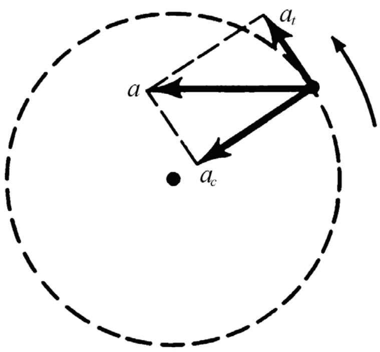

Figure 1.10

The change of the angular velocity is characterized by angular acceleration. *Angular acceleration* is called the angular velocity per unit time:

$$\varepsilon = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}$$

If the angular acceleration is constant, the circular motion is *uniformly accelerated*. Uniformly accelerated motion along a circle can be described by the following equations of motion:

$$\omega = \omega_0 \pm \varepsilon t,$$

$$\varphi = \omega_0 t \pm \frac{\varepsilon t^2}{2},$$

where ω_0 – initial angular velocity of the material point. Quite often, you want to express angular path through a number of turns. It should be remembered that in one revolution of the material point passes angular path 2π, equal therefore if you made a material point N of rotations, it means that it has passed the angular distance equal to $2\pi N$, $\varphi = 2\pi N$.

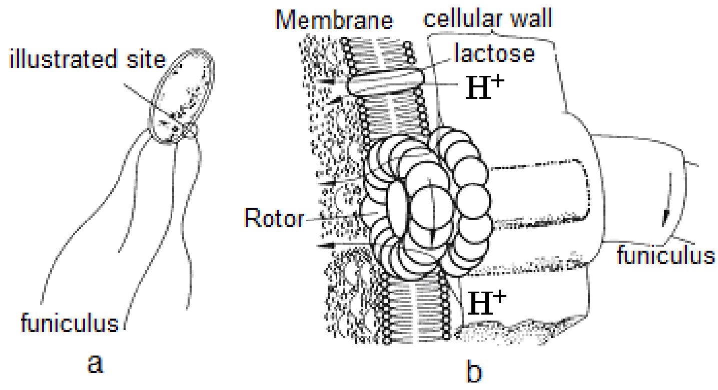

Figure 1.11

The technique of rotational movement occurs very often: the rotation of the shafts of motors and generators, wheels, turbines and propeller aircraft, centrifuges etc. n living organisms, devices like a rotating wheel can be found in many bacteria, such as Escherichia coli from attaching "motor" rotating flagella. With these bacterial flagella moves in the medium (see Figure 1.11a). The base of the flagellum is attached to the dial (Rotary) in the form of rings (see Figure 1.11b). The plane of parallel to each rotor ring fixed in the cell membrane. The rotor rotates, making up to 8 revolutions per second.

CONTROL QUESTIONS

1. What is meant by a mechanical movement?
2. What is a body called the body of reference?
3. What is the material point?
4. What kind of system is called the reference system?
5. What is a vector called the radius vector?
6. What is a trajectory?
7. What kind of motion is called rectilinear?
8. Give the definition of *speed*.
9. What is instantaneous velocity?
10. Give the definition of acceleration.
11. Please list all kinds of mechanical motion of a material point.
12. What is angular acceleration?

THE TEST TASKS

Select one or more correct answers

1. WITH ACCELERATED MATERIAL POINT MOTION ON A CIRCLE CHANGES

1) the module of the velocity
2) the direction of the velocity
3) the module and direction of the velocity
4) nothing changes

2. THE MATERIAL POINT IS …

1) the body of negligible mass
2) the body of a very small size
3) body mass which can be neglected in conditions of the tasks
4) body, dimensions which can be neglected in terms of given problem

3. THE PATH TRAVERSED BY THE MATERIAL POINT IS …

1) the amount equal to the modulus displacement a material point
2) the amount equal to the modulus of the vector from the origin to the final position of the material point
3) the amount equal to the difference vectors of modules, drawn from the origin to the initial and final position of the material point
4) the length of the trajectory of movement of the body

4. INSTANTANEOUS VELOCITY THE OF BODY IS (A) …

1) limit to which aims to average velocity of an infinitely small time interval Δt
2) the ratio of the path traveled by the body during the time interval
3) the acceleration multiplied by time
4) physical quantity that characterizes the rate of change of velocity with time

5. ACCELERATION OF THE BODY IS (A) …

1) scalar quantity equal to the velocity of the body
2) vector quantity, showing how much the velocity vector at the point of its movement per unit time
3) the quantity, equal to the force on the body mass

4) physical quantity that characterizes the rate of change, the path traveled by the body for a time interval

6. TANGENTIAL ACCELERATION BODY INDICATES

1) how rapidly velocity of the body changes in the direction of
2) how rapidly the path traversed by the body changes in modulus
3) how rapidly body changes the speed modulus
4) how rapidly will change the vector of complete acceleration

CHAPTER 2. DYNAMICS

So far we have considered the motion on the basis of the concepts of speed and acceleration. Now let us study the following questions: why do bodies move in this way rather than another? What makes a body at rest start moving? What is the cause of the acceleration or deceleration of the body? What caused the movement of the body in a circle? One could say that in each of these cases a force effects on the body. In this chapter, we examine the relationship between force and motion. In mechanics, we will consider that the cause of motion is the interaction of bodies.

Physical quantity, which is a measure of the mechanical effects on the body from other bodies or fields, called the *force*.

At the heart of classical mechanics are three laws of dynamics, formulated by Newton in 1687.

2.1. The basic laws of mechanics. Newton's Laws

<u>The first law</u> is called the law of inertia: *every body maintains a state of relative rest or uniform rectilinear motion as long as the external factors do not change this state*.

In the absence of external influences a body in motion continues to move uniformly in a straight line without changing its speed. This is the inertial motion of the body.

Through *inertia* the body maintains a state of uniform linear motion or rest if there is no action on it by other bodies. However, to test the first law by the experienced way, we are hampered by external influences (for example, Earth's gravity, and resistance to the environment surrounding the moving body.

<u>Newton's Second Law</u>, which establishes a connection between dynamic and kinematic quantities, is formulated as follows: *the acceleration* $\vec{a}$ *acquired by the body under the force* $\vec{F}$ *is proportional to this force, and inversely proportional to the mass m of the body:*

$$\vec{a} = \frac{\vec{F}}{m} \tag{2.1}$$

Because of the law, it follows that the greater the mass m, the acceleration becomes smaller for a given body under the influence of the applied force, i.e. this body is more inert. Thus, the weight characterizes the inertial properties of the body during the forward movement and is a measure of its inertia.

In practice the body can be acted on by multiple forces. However, each of these forces is independent of the other forces and tells the body acceleration determined by Newton's second law. This is the principle of superposition, according to which can be written:

$$\vec{a} = \frac{\sum_{i=1}^{n} \vec{F}_i}{m} = \frac{\vec{F}}{m}, \tag{2.2}$$

where $\vec{F} = \sum_{i=1}^{n} \vec{F}_i$ called the *resultant (or resulting)* force applied to the body (see Figure 2.1). On the basis of this principle, it becomes possible to

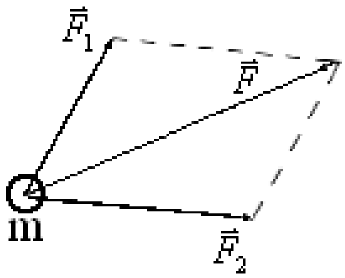

Figure 2.1

expand the force on the components and consider their effects separately.

The scalar form of the second law can be written as:

$$a = \frac{F}{m},$$

from whence

$$F = ma, \tag{2.3}$$

those acceleration is numerically equal to the product of body mass on the force.

Newton's second law can be written in another form, as

$$\vec{a} = \frac{d\vec{u}}{dt},$$

then

$$\vec{F} = m\frac{d\vec{u}}{dt}, \tag{2.4}$$

If the mass is constant, it is possible to make a differential sign, we get

$$\vec{F} = \frac{d}{dt}(m\vec{u}) \tag{2.5}$$

The vector $m\vec{u}$ is called the *momentum* or the amount of movement of the body and the same direction as the velocity vector $\vec{u}$, and $d(m\vec{u})$ expresses the change in the momentum vector. This equation is transformed to the following form:

$$\vec{F}dt = d(m\vec{u}) \tag{2.6}$$

The vector $\vec{F}dt$ is called momentum of force $\vec{F}$ acting during a short period of time dt, it has with a force in one direction. From equation (2.6), which is also an expression of the fundamental law of dynamics, should be: *change of body momentum (quantity of motion) is momentum of force acting on it.*

Newton's Third Law states that *forces with which the bodies act on each other are equal in magnitude, opposite in direction and parallel to segment connecting the centers of this bodies*, i.e.

$$\vec{F}_1 = -\vec{F}_2 \tag{2.7}$$

It should be noted that the force $\vec{F}_1$ - a force with which the second body acts on the first and applies the force to the first body, and $\vec{F}_2$ - a force with which the first body acts on the second, and attaches it to the second body, so these forces do not balance each other.

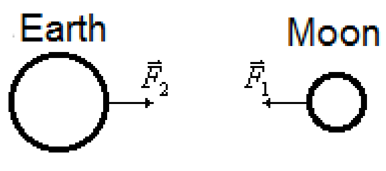

Figure 2.2

The interaction of the bodies is seen as a direct action, or action at a distance. With direct action, for example, with the impact of the hammer on the anvil, the force with which the hammer acts on the anvil, is equal in value and opposite in direction to the force which acts on the hammer anvil. An example of action at a distance is the mutual attraction of the Earth and the Sun (see Figure 2.2).

2.2 The law of conservation of momentum

The totality of the interacting N bodies is called a *system of bodies*. *External forces* - the forces acting on the part of bodies in the system of the body, but not included in it. *Internal forces* – the forces resulting from the interaction of the bodies in the system.

Closed system of bodies is a system which is not acted upon by external forces.

Consider a closed system consisting of two particles. Let the mass of the first point m_1, it speed up the interaction $\vec{u}_1$ after the interaction $\vec{u}_1'$, the speed of change du_1; respectively the second point m_2, and its velocity before $\vec{u}_2$ and after $\vec{u}_2'$ the reaction, and changes in speed du_2.
According to Newton's second law can be written:

$$\vec{F}_1 dt = d(m_1\vec{u}_1) = m_1 d\vec{u}_1,$$
$$\vec{F}_2 dt = d(m_2\vec{u}_2) = m_2 d\vec{u}_2.$$

Combining these equations, we obtain the following expression:

$$(\vec{F}_1 + \vec{F}_2)dt = m_1 d\vec{u}_1 + m_2 d\vec{u}_2.$$

Since $\vec{F}_1$ и $\vec{F}_2$ – the internal forces of the system of two material points, then by Newton's third law $\vec{F}_1 = -\vec{F}_2$. Then the expression $(\vec{F}_1 + \vec{F}_2)dt = 0$, consequently, $m_1 d\vec{u}_1 + m_2 d\vec{u}_2 = 0$ or $d(m_1\vec{u}_1 + m_2\vec{u}_2) = 0$.

Since a constant differential is zero, that

$$m_1\vec{u}_1 + m_2\vec{u}_2 = \mathrm{const} \qquad (2.8)$$

The last equation expresses the law of conservation of momentum, which is formulated as follows: *the total momentum of closed system of bodies is a constant, i.e., it does not change with time and the internal forces of the system cannot change the total momentum of a closed system.*

Therefore, if the momentum of one of the bodies in a closed system changed, it could only have been due to changes in the momentum of other bodies of the system. Please note that the law of conservation of momentum - this is a vector law.

2.3 Different types of forces in mechanics

In the study of mechanical processes various forces are considered that differ by their origin.

Gravitational interaction. The law of gravity is formulated as follows: any two material points interacting with a force proportional to the product of their masses (m_1 and m_2*) and* inversely proportional to the square of the distance between them:

$$F = \gamma \frac{m_1 m_2}{r^2}. \tag{2.9}$$

The coefficient of proportionality is called the gravitational constant. It is experimentally found that the numerical value of the gravitational constant is equal to $\gamma = 6{,}67 \times 10^{-11}\,\text{м}^3/(\text{кг}\cdot\text{с}^2)$.

Each body of mass located on the Earth is attracted to the Earth by the force directed to the center and its equal:

$$F = \gamma \frac{mM}{R^2},$$

where M - the Earth's mass R - radius of the Earth, if the body is near Earth's surface. This force is called gravity and symbolically written as: $F = mg$ which g is called the acceleration of free fall $g = \gamma \frac{M}{R^2}$. From this formula it follows that the gravitational acceleration does not depend on the mass and size of the bodies near the Earth's surface.

Elastic forces. Under force a body may be deformed, i.e. one part can be displaced relative to each other. This occurs within the deformed body when the reaction force is equal in magnitude to the deforming force and is called an elastic force. Experience shows that the magnitude of the elastic force generated by the small displacements from equilibrium position and displacement Δx *are proportional to each other, i.e.*

$$F = -k\Delta x, \tag{2.10}$$

where k - coefficient of elasticity, depending on the properties of the material body. This relationship is called Hooke's law. The minus sign indicates the opposite direction of the elastic force and displacement.

As can be seen a vertically extending spring, one end of which is fixed, and the suspended load to another exemplary elastic body. If you release the load, the spring is stretched - its length is increased by value Δx (see Figure 2.3). $\Delta x = x_2 - x_1$.

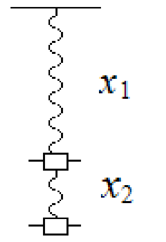

Figure 2.3

In the spring at the same time there is an elastic force F tending to return the load to the original position. The elastic force is always directed to the position of equilibrium of the body.

Frictional forces – these forces are caused by the interaction of molecules in interacting bodies. The friction forces are directed along the contact surfaces in such way that they counteract a relative displacement of these surfaces.

The force of friction that occurs when sliding surfaces relative to each other, independent of the surface area of friction bodies contact and is proportional to the normal pressure force, pressing the friction surfaces to each other:

$$F_{тр} = \mu F_n, \tag{2.11}$$

where F_n is normal component of the gravitation force, μ is friction constant. When there is movement of solids in liquids and gases there are friction forces depending on the shape and size of the body, the state of its surface, speed with respect to the environment and from properties of the medium, which are called the *viscosity*. At low speeds, the friction force increases linearly with speed

$$F_{тр} = \eta_1 u, \tag{2.12}$$

at high speeds – its square

$$F_{тр} = \eta_2 u^2 \tag{2.13}$$

2.4 Work done by a constant force

In physics, work has a strictly defined meaning. If we apply force to a body and move it to a certain distance, then we say that *work* is done.

Work done by a constant (both in value and direction) force when moving the body, determined as

$$A = FS\cos\alpha = \left(\vec{F} \times \vec{S}\right), \tag{2.14}$$

F – constant force, S – the resulting motion, α – the angle between the force and displacement (see Figure 2.4).

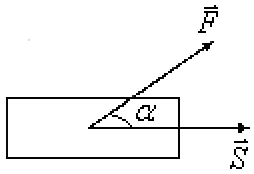

Figure 2.4

Work is a scalar quantity. If the force vector and the direction of motion form an acute angle α ($\cos\alpha > 0$) the work is considered positive. If the angle α - dull ($\cos\alpha < 0$) the work is negative. Force can be applied to the body but does not perform work. For example, if you are holding a heavy bag of groceries and do not move, then you are not doing the work; you can get tired (and really your muscles consume energy), but because the bag is at rest (i.e., its displacement is zero), work $A = 0$. No work is done and provided that the angle between the direction of the force and displacement is equal $\frac{\pi}{2}\left(\alpha = \frac{\pi}{2}\right)$, i.e. the force acting perpendicular to the body movement, then work is not produced.

2.5 Work done by a variable force

In many cases, the process of moving force varies in magnitude and direction. For example, the material moves along the path shown in Figure 2.5, at the same time in different parts of the trajectory of the force acting on a material point is different in direction and magnitude.

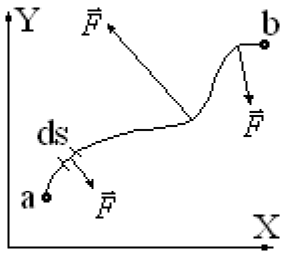

Figure 2.5

To find done work in this case, you need the whole path divided into infinitesimal straight portions ds, which force can be considered constant. At each of these small areas is performed work $dA = Fds\cos\alpha$, where α is the angle between the force vector $\vec{F}$ and the infinitesimal displacement $d\vec{s}$ at a given point of the trajectory. Since the work is a scalar value, then to find the work done in the final stage of the journey s, we need to sum up the work done on the sites ds or to integrate expression $dA = Fds\cos\alpha$. In carrying out this operation, we obtain

$$A = \int_a^b F \cos\alpha ds \tag{2.15}$$

The unit of work is the joule (J). Joule is the work of driving force in one newton on the segment path of 1 meter:

1J=1N·1m

Entering the concept of power is performed to characterize the speed of work doing.

Power is a physical quantity that is numerically equal to the work done per unit of time:

$$N = \frac{dA}{dt} = Fu\cos\alpha \tag{2.16}$$

Unit power – 1 watt (W). It is easy to show that 1 watt = 1 joule/sec.

2.6 Energy

The body's ability to do work in mechanics is called energy.

Changing the energy system of bodies has always valued the work done by external forces applied to the system:

$$\Delta W = W_2 - W_1 = A \tag{2.17}$$

Energy is measured in the same units as done work.

When the work of external forces is positive ($A > 0$), then the energy of the system increases and, conversely, if the work of the external forces is negative (the system does work), the energy of the system decreases. Consequently, the system can perform work only by changing their energy.

There are two types of mechanical energy:

1) the kinetic energy - an energy possessed by the moving body;

2) the potential energy - an energy that characterizes the interaction of bodies or parts of the body.

2.7 Kinetic energy

The moving body can do the work of another body which it impinges, for example: moving a hammer to drive nails, flying cannonball smashes the wall, etc. Therefore, we say that a moving body has kinetic energy. In order to obtain a quantitative expression for the kinetic energy, let's calculate the work that can make a moving body in the particular case of one-dimensional motion. Let the body mass m_1 moves along a straight line at a speed u_1 under the force F. We find an expression of the work force:

$$A = \int_a^b F\cos\alpha\, ds,$$

because the force F changes the velocity of the body, it receives him the acceleration, therefore according to the 2nd Newton's law $F = m_1 a$, we substitute this force's value to the expression for work: $A = \int_a^b F\cos\alpha\, ds = \int_a^b m_1 a\cos\alpha\, ds$, but $a\cos\alpha$ equal to the tangential acceleration of the body, as the direction coincides with the direction of the velocity, so $a\cos\alpha = a_\tau = \dfrac{du}{dt}$. Then we can write an expression as following

$$\begin{aligned} A &= \int_a^b F\cos\alpha\, ds = \int_a^b m_1 a\cos\alpha\, ds = \int_a^b m_1 \frac{du}{dt} ds = \int_{u_{нач.}}^{u_{кон.}} m_1 u\, du = \\ &= \frac{1}{2} m_1 u^2 \Big|_{u_{нач.}}^{u_{кон.}} = \frac{1}{2} m_1 u_{кон.}^2 - \frac{1}{2} m_1 u_{нач.}^2 \end{aligned} \tag{2.18}$$

The value $E_k = \dfrac{1}{2} m u^2$ is called *kinetic energy*. Thus, the work is numerically equal to changing of kinetic energy of the body, i.e. the work done on the body, always changes its kinetic energy:

$$A = E_{кон.} - E_{нач.} = \Delta E \tag{2.19}$$

On the other hand, we can say that if you change the kinetic energy of the body, the body does work.

2.8 Conservative forces

Force can be divided into two classes: the conservative and non-conservative. Any force is called conservative if:

a) it depends only on the position of the body on which acts;

b) work produced by the force on the particle, traveling between any two points in space, depends only on the initial and final positions of the particle and is therefore not dependent on the shape of its trajectory.

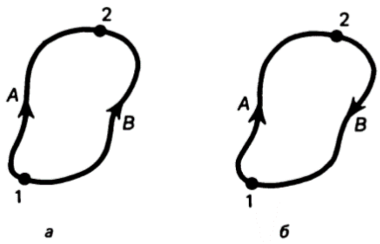

Figure 2.6

Figure 2.6a - particle moves from point 1 to point 2 in two different paths A and B; Figure 2.6b - a particle moves in a closed circuit from point 1 to point 2 on the path A and back from point 1 to point 2 along path B.

There is another equivalent definition of a conservative force: it is a force, which works on the body with its motion along any closed path, when the body returns to its original position, is always zero. To prove this, let's consider a particle moves from point 1 to point 2 along two paths, designated A and B in Figure 2.6a. If we assume that the particle is acted conservative force, according to the first definition, the work done by it when moving along the path of the particle A is the same as the path B. Let's designate this work on the movement of particles from point 1 to point 2 by W. Now let the particle moves along a closed path (see Figure 2.6b). From point 1 to point 2 particle moves along path A, and the force does work W. Further particle returns from point 1 to point 2 on the way B. What kind of work does with the force? When you move the particle from point 1 to point 2 on the way B work is producing and it is by definition equal $\int FdS$. In the reverse movement of the particle from point 1 to point 2 the force F at each point is the same as when moving from point 1 to point 2, but the direction dS is reversed. Hence, at each point of the product FdS has the opposite sign, i.e. work on the way back from a point 2 to point 1 is $(-W)$. Thus, the total work done in movement the particle from point 1 to point 2 and back is $W+(-W)=0$ what proves the determination of conservative forces.

For non-conservative forces include frictional force. The work done when moving a heavy box along the horizontal floor, is the product of the frictional force on the full path traversed compartment, since the frictional force at each point of the trajectory is directed precisely against the motion. Therefore, the work of friction forces when moving the body from one point to another along the straight line joining these points is less than the work done when the body is moved along a curved path, such as a semicircle.

Conservative force is the gravitation force and the force of elasticity. Consider the work done by gravity.

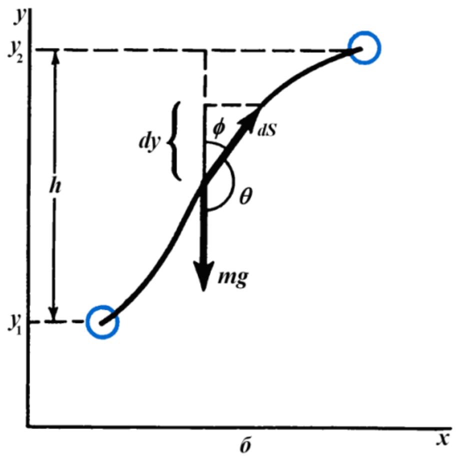

Figure 2.7

Assume that the body moves along a trajectory in an arbitrary plane xy (see Figure 2.7b). It starts the movement of the point with the vertical coordinate y_1 (height), and reaches the height y_2, the $y_2 - y_1 = h$. With the help of formula (2.15) we calculate the work A_g done in this process is the gravitation force:

$$A_g = \int_1^2 F_g dS = \int_1^2 mg\cos\theta dS$$

Denote $\phi = 180° - \theta$ the angle between the motion vector dS and the vertical component dy, as shown in Figure 2.6b. Given that $\cos\theta = -\cos\phi$ and $dy = dS\cos\phi$, we find

$$A_g = -\int_{y_1}^{y_2} mgdy = -mg(y_2 - y_1)$$

Because $y_2 - y_1 = h$ is vertical height, we see that the work depends only on the height and does not depend on the particular selected trajectory.

2.9 Potential energy

The potential energy is related to the mutual arrangement of the interacting bodies, and also as the kinetic energy can be measured by the work, which can do the system moving from one state to another. Since the potential energy reflects the interaction of the bodies or parts of bodies, in the calculation we should take into account the nature of the work force, due to which the interaction. Therefore, quantification of the potential energy for the various forces will be different.

Let's find an expression for the elastic potential energy of the compressed (elastically extended) spring. In this case, the potential energy depends on the relative position of individual body parts (e.g., the distance between adjacent turns of the spring). In order to find a potential energy of elastic compression of the spring, it is necessary to calculate the work that has been spent on the accumulation of this energy. Since the spring force depends on the degree of compression or stretching $F = -k\,dx$, than the expression for work of body's movement $ds = dx$ can be written:

$$A = \int F \cos\alpha ds = \int_{x_2}^{x_1} -kxdx. \tag{2.20}$$

Integrating this expression from an initial value x_2 to a final value x_1 of stretching, we get

$$A = \int_{x_2}^{x_1} -kxdx = \left(\frac{kx_1^2}{2} - \frac{kx_2^2}{2}\right) = -(U_2 - U_1), \tag{2.21}$$

where $U = \frac{kx^2}{2}$ is an expression of the potential energy of the spring.

Bodies raised above the Earth's surface have the potential energy. To raise the body with weight m to height h it is necessary to perform work (provided that $u = \text{const}$)

$$A = F \times s\cos\alpha = mgh, \tag{2.22}$$

this work will go to increasing the energy system of the body- Earth, i.e.

$$A = -(U_2 - U_1)$$

Considering that in a state where the body was on the Earth's surface, the potential energy of the system $U_1 = 0$, we obtain $A = U_2$. Thus, the potential energy of the body, raised to a height h above the Earth's surface, is equal to

$$U = mgh \tag{2.23}$$

In the derivation of this formula it was assumed that the gravitation acceleration is constant.

It should be noted that the value of the potential energy dependent on the choice of the zero level, i.e. level at which the potential energy is equal to zero.

In the calculation of the elastic potential energy of the spring was considered the work done by the internal forces of the system. Therefore, in determining the potential energy of the body raised above the Earth and it is advisable to go to the internal forces of the system. When the body is elevated above the ground motion velocity of a body is constant, then the external force $F=(-mg)$, i.e. for internal forces of system (in this case, the internal force is the gravitation force), the performed work will be negative.

Thus, the work done by the internal forces of the system, the changing of potential energy related as follows:

$$A=-(U_2-U_1)=-\Delta U \tag{2.24}$$

2.10 Law of energy conservation

To derive the law of conservation of energy we consider the simplest case of free fall of the body. In this case, the system consists of two bodies: the falling body and the Earth. It will act only gravity mg on the body. Under the influence of this force the body acquires the acceleration, i.e. its falling speed as it approaches the Earth will increase, which means that gravitation force does the work that goes on the change in the kinetic energy of the body.

$$A=\Delta E \tag{2.25}$$

Also, due to the gravitation force changes the relative position of the body relative to the Earth, i.e. it is changing the potential energy of the body. According to (2.24)

$$A=-\Delta U \tag{2.26}$$

Since the work accomplished to the same force, then we can write the following equation

$$\Delta E=-\Delta U \tag{2.27}$$

or $$\Delta(E+U)=0, \tag{2.28}$$

from whence $$W=E+U=\mathrm{const} \tag{2.29}$$

The value W equal to the sum of kinetic and potential energy is called a complete mechanical energy, or just body's energy. This relation (2.29) expresses the law of conservation of energy of the body. This law is formulated as follows: *when a body moves under the influence of the conservative forces kinetic and potential energies transformed into each other, but their sum remains constant in an isolated system of bodies.*

CONTROL QUESTIONS

1. Identify the basic laws of mechanics.
2. What is the law of conservation of momentum?
3. Formulate the law of universal gravitation.
4. Define the operation, write the formula and units.
5. Give the definition of energy?
6. The kinetic energy of this?
7. The potential energy of this?
8. The law of conservation of energy.

THE TEST TASKS

Select one or more correct answers.

1. INERTIA IS (A) …

    1) measure the body inertia in the rotational motion
    2) phenomenon in which the body maintains a state of uniform linear motion or rest, if there is no action on it of other bodies
    3) phenomenon in which the body maintains a state of uniform linear motion by the force
    4) a phenomenon in which the body maintains a state of uniform motion accelerated by the force

2. NEWTON'S FIRST LAW FORMULATES

    1) every body does not save the state of relative rest or uniform rectilinear motion
    2) every body maintains a state of uniform rectilinear motion by the force
    3) every body maintains a state of relative rest or uniform rectilinear motion as long as the external factors will not change this state.
    4) every body maintains a state of relative calm under the influence of gravity

3. INERTIAL REFERENCE SYSTEM IS THE SYSTEM

    1) in which the body moves without acceleration
    2) in which the body is accelerated only by the action of other bodies
    3) which moves at a velocity much less than the velocity of light
    4) which moves with a velocity equal to velocity of light

4. NEWTON'S SECOND LAW IS FORMULATED

    1) acceleration, the body acquired by the force proportional to this force and inversely proportional to the mass of the body
    2) acceleration, the body acquired by the force in proportion to this force and the weight of the body
    3) acceleration, the body acquired by the force proportional to the mass of the body and inversely proportional to this force
    4) acceleration, the body acquired by the force proportional to the square of this force and inversely proportional to the mass of the body

5. IMPULSE BODY IS (A) …

1) vector physical quantity characterizing the measure of the mechanical movement of the body and equal to the product of the weight by the square of its velocity
2) the scalar value that characterizes the amount of mechanical movement of the body and is equal to the product of body mass on his velocity
3) vector physical quantity characterizing the measure of the mechanical movement of the body and is equal to the product of body mass on his velocity
4) scalar quantity, characterizing the amount of mechanical movement of the body and is equal to the product of the square of the body weight on his velocity

6. BODY IS AT REST ON AN INCLINED PLANE. THE FRICTIONAL FORCE ACTING ON THE BODY, HEADING

1) upward along a plane
2) the friction force is equal to zero
3) along the plane down
4) parallel to the plane

CHAPTER 3. MECHANICAL OSCILLATIONS AND WAVES

Many bodies are able to fluctuate or oscillate: cargo at the end of a spring, a tuning fork, wheel balancer hours, pendulum, plastic ruler firmly pressed against one end to the edge of the table, strings of a guitar or piano. Spiders discover food lodged in their web, the vehicle body oscillates up and down on its springs when the vehicle passes bumps, houses and bridges shaking when heavy trucks drive by and even in strong winds. *Almost all of the material things vary (at least briefly) after they are affected by the momentum of force.* At the atomic level, the atoms of a solid oscillate about their fixed positions in the lattice.

In the general case, *vibrational processes or oscillations are processes that are exactly or approximately repeated at identical intervals of time.*

Free oscillations, or natural oscillations, are those oscillations that occur in a system under the influence of the internal forces of this system after the system has been taken out of equilibrium. The resultant internal force, under the influence of which the oscillatory process occurs, is called the returning force, because it seeks to return to the position of equilibrium a body or a material point deviated from this position. Natural oscillations are the most common in the theory of oscillatory processes.

Forced referred to fluctuations that occur under the influence of periodically changing external forces. Terms of occurrence and nature of forced oscillations depend essentially on the nature of the natural oscillations.

3.1. Harmonic oscillations

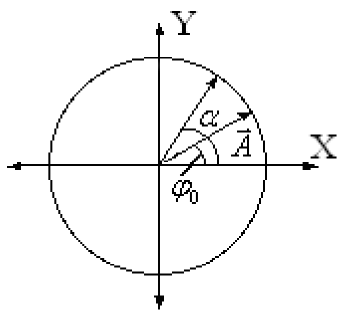

Figure 3.1

An example of the most basic natural oscillations is harmonic oscillations.

Harmonic oscillations are periodic processes in which the magnitude of the change is the law of the sine (or cosine). Let the motion of a material point is described by the radius vector $\vec{A}$ and let the point makes a uniform circular motion with an angular velocity of rotation ω (see Figure 3.1). Then the projection of the radius vector $\vec{A}$ on the axes x and y can be written as follows:

$$\begin{aligned} x &= A\sin\alpha = A\sin(\omega t + \varphi_0) \\ y &= A\cos\alpha = A\cos(\omega t + \varphi_0) \end{aligned} \qquad (3.1)$$

Thus, the change vector $\vec{A}$ of the projections on the axes x and y occurs under the laws of the sine and cosine. Therefore, the circumferential movement is a harmonic oscillatory motion.

In (3.1) the values of x and y are called *the displacement*. The offset is the distance from the point of the oscillating equilibrium position at any given time.

The highest point of the oscillating displacement from the equilibrium position is called *the amplitude* of oscillations in the expression (3.1) – is the value A.

During one turn of the oscillating point it will return to its original position and the projection of the radius vector will make one complete oscillation.

The period of oscillation (T) is the time during which the material point makes a one complete oscillation.

The oscillation frequency (ν) is the number of complete oscillations committed per unit of time. Period and frequency of the oscillations are related as follows:

$$\nu = \frac{1}{T},$$

$$\omega = 2\pi\nu,$$

where ω – circular (or cyclic) frequency harmonic oscillations.

Cyclic oscillation frequency is related to the period of oscillation and the frequency of

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

The frequency ν is measured in *hertz*, the dimension [Hz] = 1/sec.

The variable $\omega t + \varphi_0$ is an argument of the sine and cosine, and called *the phase fluctuations*; parameter φ_0 is called *the initial phase*. The initial phase shows the position of the vibrating point at the initial time.

3.2. Velocity and acceleration of harmonic oscillations

Let the displacement of the oscillating point is determined by the law

$$x = A\sin(\omega t + \varphi_0) \qquad (3.1)$$

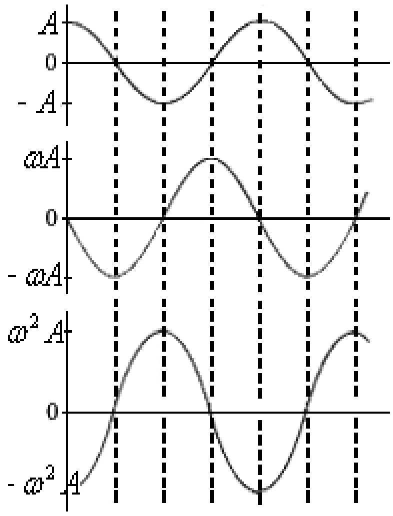

Figure 3.2

Then, according to the laws of mechanics, the velocity of this point is determined by the first derivative of displacement in time:

$$u = \frac{dx}{dt} = wAcos(wt + j_0) = wAsin(wt + j_0 + \frac{p}{2}), \qquad (3.2)$$

those velocities vary harmonically, ahead of the displacement x of phase π/2. In the position of equilibrium velocity of a material point reaches its maximum u_{max} = A ω.

Acceleration, as well as the mechanics is determined by the first derivative of the velocity in time:

$$a = \frac{du}{dt} = - w^2 A\sin(wt + j_0) = Aw^2\sin(wt + j_0 + p) \qquad (3.3)$$

and as well as the velocity varies harmonically ahead displacement in phase by π.

Graph displacement, velocity and acceleration of the harmonic oscillator shown in Figure 3.2. Note that the velocity is different in phase from the displacement on the π/2 and acceleration - to π.

3.3. Fluctuations in of the spring

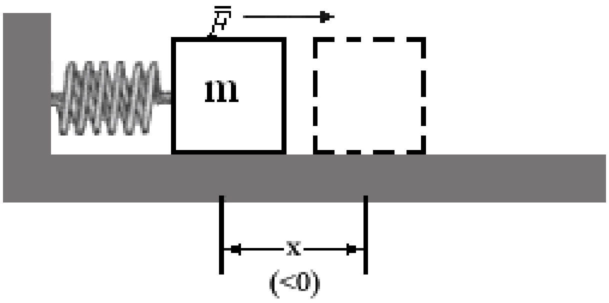

Figure 3.3

Consider load fluctuations at the end of the spring. Many other types of oscillatory movements show great similarity with these fluctuations; for example, the vibrations occurring in the circulatory system, breathing, muscle contraction.

We assume that we can neglect the mass of the spring and that the spring is mounted horizontally, as shown in Figure 3.3.

The load mass *m*, which moves without friction on a horizontal surface, is attached to one end of the spring. Every spring has a certain length value at which its part of the cargo is not a force; In this case we say that the spring is in an *equilibrium position.*

If you move the cargo to the right, stretching the spring or to the left, squeezing it, the spring acts on the load with a force that tends to return it to a position of equilibrium; a force called the returning. For our system the restoring force is directly proportional to the distance *x*, at which the spring is compressed or stretched (see Figure 3.3):

$$F = -kx \tag{3.4}$$

The minus sign means that the restoring force is always opposite to the direction of displacement *x*. If in Figure 3.3, we will send the axis, for example, to the right, note that the position of equilibrium, we have chosen at the point *x* = 0. When the spring is compressed, force is directed to the right (see Figure 3.3.), and the movement of *x* to the left side. The constant k in equation (3.4) is *the spring constant*.

What happens if the spring stretched to a length x = A, and then let go? The spring acts on the load with a force that tends to return it to its equilibrium position. But because of this power according to the load acceleration, the weight comes to a position of equilibrium at considerable velocity. Note that in equilibrium the force acting on the load is reduced to zero and its velocity at this point is at its maximum (see Figure 3.2). When the load slipped equilibrium position moves to the left, the force from the spring slows it down, and at x = -A the load stops momentarily and then starts to move in the opposite direction until you come to the point $x = A$, where he started the movement i.e. the load made one complete oscillation. Then the whole process repeats.

The equation of Newton's second law for the load on the spring is:

$$m\frac{d^2x}{dt^2} = -kx$$

Let us transform this equation as follows:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \tag{3.5}$$

The coefficient of the displacement is positive, so it can be represented as follows:

$$\omega_0^2 = \frac{k}{m} \tag{3.6}$$

Using equation (3.5) the notation (3.6), we obtain:

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \tag{3.7}$$

Thus, the movement of the load by the force of the form (3.4) is described by the linear homogeneous second order differential equation. It is easy to see that the general solution of equation (3.7) has the form:

$$y = A\cos(\omega_0 t + \varphi_0) \tag{3.8}$$

Displacement x varies with time according to the cosine law. Consequently, the motion of the system under the action of force F = -kx, is a harmonic oscillation. From equation (3.8) that the introduced the coefficient is the frequency of oscillation is called the natural frequency of oscillation of the system, is given by:

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{3.9}$$

3.4. The total energy of harmonic oscillations of its own

Let us find out how to change over time, the kinetic and potential energy of the harmonic oscillation on the example of an elastic spring. Kinetic energy is equal to

$$E_к = \frac{1}{2}mu^2 = \frac{mA^2w_0^2}{2}\cos^2(w_0t + j_0) \qquad (3.10)$$

The potential energy is given by

$$U = \frac{kx^2}{2} = \frac{kA^2}{2}\sin^2(\omega_0 t + \varphi_0) \qquad (3.11)$$

Combining (3.10) and (3.11), taking into account the relation (3.6), we obtain:

$$W = E_k + U = \frac{mA^2w_0^2}{2}\cos^2(w_0t + j_0) + \frac{kA^2}{2}sin^2(w_0t + j_0) \quad (3.12)$$

$$= \frac{mA^2w_0^2\cos^2(w_0t + j_0)}{2} + \frac{w_0^2mA^2sin^2(w_0t + j_0)}{2}$$

$$= \frac{mA^2w_0^2}{2}\left(\cos^2(w_0t + j_0) + sin^2(w_0t + j_0)\right) = \frac{mA^2w_0^2}{2}$$

$$= \frac{kA^2}{2}$$

The relation (3.12) shows that the total energy of free oscillations is equal to the maximum potential energy and maximum kinetic energy of the harmonic vibrations and is directly proportional to the mass of the oscillating point m, the square of the amplitude A^2 and the square of the frequency of oscillation.

3.5. Addition of oscillations directed along the same line

There may be cases when the several elastic forces affect a body. Each of these forces causes the body to make a harmonic oscillation. With simultaneous exposure to these forces at the same time the body will be involved in all of these movements. An example is the eardrum, which simultaneously perceives plurality sound oscillations. In this case, to find the resulting motion, the oscillations must be added.

Let’s consider the case when the body is simultaneously involved in two oscillations with the same frequencies but with different amplitudes and initial phases:

$$x_1 = A_1 \cos(\omega t + \varphi_{01}),$$

$$x_2 = A_2 \cos(\omega t + \varphi_{01}).$$

The resulting oscillation is also a harmonic and is the sum of the displacements

$$x = x_1 + x_2$$

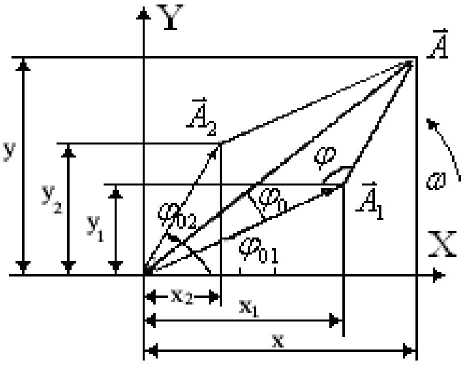

Figure 3.4

Vector method of oscillations addition is used to determine the resulting displacement (see Figure 3.4) because any harmonic oscillation can be described by the radius-vector, the module of which is equal to the amplitude of the oscillations (see Section 3.1).

The angle φ is measured from the positive direction of the x-axis. Figure 3.4 shows the positions of the vectors $\vec{A}_1$ and $\vec{A}_2$ at the initial time $t = 0$ since vectors rotate at the same angular velocity, the resulting vector $\vec{A}$ will rotate with the same angular velocity, i.e. resulting movement will also be harmonic with the circular frequency ω:

$$x = A\cos(\omega t + \varphi_0).$$

The amplitude of this oscillation can be found on the cosine theorem:

$$A^2 = A_1^2 + A_2^2 - 2A_1A_2 \cos\gamma,$$

$$\cos\gamma = \cos[\pi - (\varphi_{02} - \varphi_{01})] = -\cos(\varphi_{02} - \varphi_{01}), \text{ so}$$

$$A = \sqrt{A_1^2 + A_2^2 - 2A_1A_2 \cos\gamma} \qquad (3.13)$$

The initial phase φ_0 of the resulting oscillation can be determined from the initial conditions:

$$tg\varphi_0 = \frac{y}{x} = \frac{y_1 + y_2}{x_1 + x_2} = \frac{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}}{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}} \qquad (3.14)$$

Analyzing the equation (3.13) we see that with the addition of oscillations with the same direction are possible the following cases:

1. If the phase difference is an even number π, i.e.

$$\varphi_{02} - \varphi_{01} = 2k\pi ,$$

where $k = 0,1,2,...$ (we can assume that $\varphi_{02} - \varphi_{01} = 0$), the oscillations are in phase and reinforce each other. In this case

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2} = \sqrt{(A_1 + A_2)^2} ,$$

$$A = A_1 + A_2 ,$$

those the resulting amplitude is equal to the sum of amplitudes of the

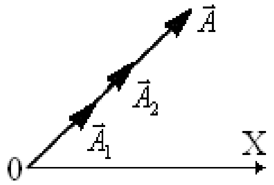

Figure 3.5

oscillations (see Figure 3.5).

2. When the phase difference equal to an odd number π

$$\varphi_{02} - \varphi_{01} = (2k+1)\pi , \ \cos(2k+1)\pi = -1,$$

$$A = \sqrt{A_1^2 + A_2^2 - 2A_1A_2} ,$$

$$A = A_1 - A_2 , \tag{3.15}$$

Those oscillations are weakening each other and the resulting amplitude is equal to the difference between the amplitudes of the folded oscillations (see Figure 3.6). If that is $A = 0$, i.e. if the amplitudes of the folded oscillations are the same and oscillations are committed in the opposite phase, then the oscillations extinguish each other.

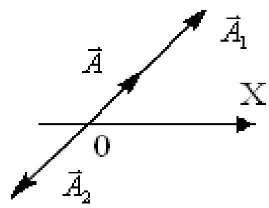

Figure 3.6

3. If foldable oscillations have the same amplitude ($A_1 = A_2$), but the frequencies of folded oscillations are not the same, the resulting oscillation is not harmonic.

Of particular interest is the case when two folded harmonic oscillations of the same directions do not differ in frequency $\omega_1 \approx \omega_2$. The resulting oscillation under these conditions can be regarded as a harmonic oscillation with pulsating amplitude (see Figure 3.7). Such oscillations are called *beats*. Schedule changes in amplitude over time are shown in Figure 3.7b.

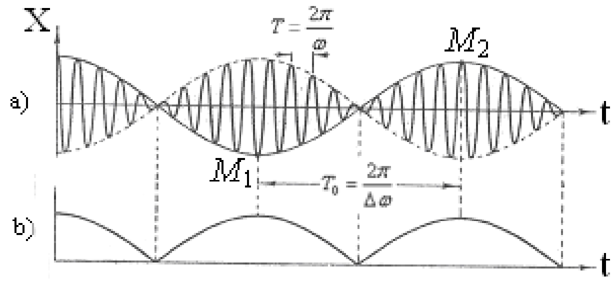

Figure 3.7

3.6. Damped oscillations

In real conditions the friction forces, which oppose the movement, effect from the environmental to the body. It is expended the energy on overcoming the forces of friction. Therefore, the oscillating energy of the body is reduced and, consequently, decreased the amplitude of the oscillations, i.e., oscillations are damped (see an equation 3.12).

Let's write Newton's second law for the real conditions:

$$\vec{F}_{\text{упр}} + \vec{F}_{\text{тр}} = m\vec{a},$$

because all the forces act along a single line, the scalar form of motion equation will be: $F_{\text{упр}} - F_{\text{тр}} = ma$.

The friction force is proportional to the speed and directed opposite to it at low speeds of movement, so

$$F_{\text{тр}} = -\mu u = -\mu \frac{dx}{dt},$$

where μ – *constant of friction*. Then Newton's second law can be written:

$$m\frac{d^2x}{dt^2} = -kx - \mu x$$

transfer all terms of the equation to the left side and divide by *m*:

$$\frac{d^2x}{dt^2} + \frac{\mu}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 \qquad (3.16)$$

We obtain the differential equation of damped oscillations, the general solution of which will be as follows:

$$A = A_0 e^{-\alpha t}\cos(\omega t + \varphi_0), \qquad (3.17)$$

where ω – the circular frequency the damped oscillations:

$$\omega^2 = \omega_0^2 - \alpha^2 \qquad (3.18)$$

Expression (3.17) is the equation of damped oscillations. It differs from the harmonic oscillations in that the amplitude of the oscillations $A = A_0 e^{-\alpha t}$ decreases with time (see Figure 3.8). The dotted line in this figure shows the amplitude dependence of the time.

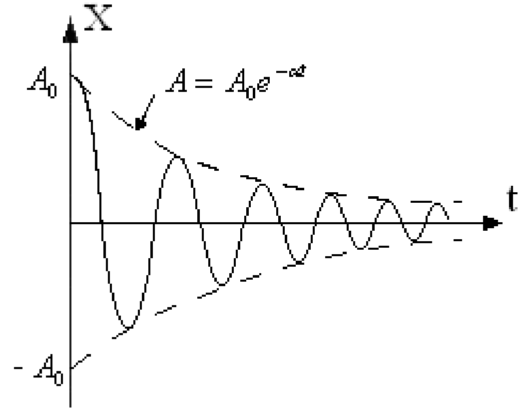

Figure 3.8

$\alpha = \dfrac{\mu}{2m}$ is called the *constant of attenuation*: the larger the attenuation constant, the faster the oscillations are damped. In practice, the degree of attenuation is characterized by a logarithmic decrement. *The logarithmic decrement* λ is the natural logarithm of the ratio of two amplitudes of the damped oscillations, differing by a single period:

$$\lambda = \ln\frac{A(t)}{A(t+T)} = \ln\frac{A_0 e^{-\alpha t}}{A_0 e^{-\alpha(t+T)}} = \ln e^{\alpha T} = \alpha T$$

The physical meaning of the logarithmic decrement is that it is the inverse of the number of oscillations after whose amplitude is decreasing in e times:

$$\lambda = \frac{1}{N_e},$$

where N_e is the number of oscillations which an oscillating body can perform for the time during the oscillation amplitude decreases in e times.

With increasing of friction oscillation frequency $\omega = \sqrt{\omega_0^2 - \alpha^2}$ decreases, and the period $T = \dfrac{2\pi}{\omega}$ respectively is increases. In case of equal of the attenuation constant α corresponds to the natural oscillations frequency ($\alpha = \omega_0$), the frequency of damped oscillations $\omega = 0$, and $T \to \infty$, the motion becomes aperiodic, i.e., with a large friction of body, brought out of equilibrium, then it slowly return to equilibrium and the oscillations do not occur.

3.7. Forced oscillations

We call oscillations *Forced* when they occur in the system under the action of a permanent external force which changes according to a periodic law, such as

$$F = F_0 \cos(\Omega t + \varphi), \tag{3.19}$$

where F_0 is the amplitude of the driving force, Ω is the frequency with which the driving force is changed. Then Newton's second law can be written as

$$\vec{F}_{упр} + \vec{F}_{тр} + \vec{F} = m\vec{a}$$

In scalar form $F_{упр} + F_{тр} + F = ma$. Let the friction force is equal to zero $F_{тр} = 0$, then

$$m\frac{d^2x}{dt^2} = -kx + F_0 \cos(\Omega t + \varphi), \tag{3.20}$$

dividing an equation (3.20) on m , and transferring members with x in the left-hand side, we obtain the differential equation,

$$\frac{d^2x}{dt^2} + \omega x = F_0 \cos(\Omega t + \varphi) \tag{3.21}$$

This is an inhomogeneous differential equation of 2nd order for forced oscillations, so its solution has the form

$$x = A\cos(\Omega t + \varphi) \tag{3.22}$$

The body will oscillate with the frequency of the driving force by the action of the driving force. The value of the amplitude in the equation (3.22) is defined as follows:

$$A = \frac{F_0}{m(\omega_0^2 - \Omega^2)}$$

We see that the amplitude of the oscillation is dependent not only of the amplitude of the driving force, but also the difference of the squares of the natural frequency and the frequency of the driving force. Graphically this dependence is shown in Figure 3.9. Separate curves on the graph correspond to different parameter values α (attenuation constant), the

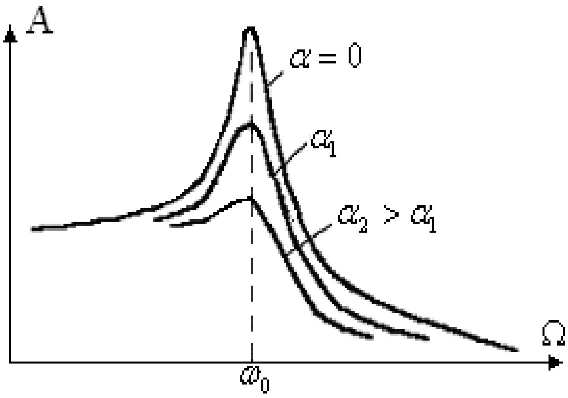

Figure 3.9

lower attenuation constant α, the higher and to the right is the maximum of the curve.

In case of equal driving force frequency and natural oscillations frequency $\Omega = \omega_0$, the oscillation amplitude increases sharply, and tends to infinity $A \to \infty$. The phenomenon of the sharp increase of the forced oscillation amplitude when the driving forces frequency coincidences with the natural frequency of the system is called *resonance*. In real conditions, the presence of friction limits the increasing of the amplitude. With increasing of friction the oscillation amplitude and the resonance frequency are decreasing.

Resonance can be both useful and harmful phenomenon. The harmful effects of resonance due to the changes that it can cause, for example, the effect of infrasound to the human internal organs. On the other hand, the resonance can detect even very weak oscillations.

3.8. Mechanical waves

The process of wave propagation in the medium is called a *wave*. The spread of mechanical oscillations in the medium is due to the presence of force relations in the matter. The particles of the medium in which the wave propagates are not transferred by the wave and only oscillated about it equilibrium positions. Depending on the direction of oscillations of the particles with respect to direction in which the wave propagates, it is distinguished by *longitudinal* and *transverse* waves. In a longitudinal wave, particles of the medium oscillate along the direction of wave propagation, so most of the longitudinal waves are spread of compression or tension (e.g., springs), compression or depression. In the transverse wave the particles of the medium oscillate in directions perpendicular to the direction of wave propagation (for example, if to pull anchored one end of a rope, the transverse waves will propagate through it).

Spreading from the oscillation source, the wave process covers more and more of the space. The locus of points, which oscillations are reaching up to this time, is called the *wave front*. Thus, the wave front is the surface that separates the space portion already involved in a wave process. The wave front is moving the whole time in the space, while remaining perpendicular to the wave propagation.

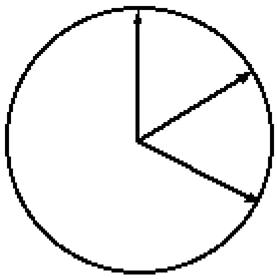

Figure 3.10

Let's take a point source of oscillations and consider the spread of oscillations caused by this source in a homogeneous, isotropic medium. Obviously, if the oscillations will propagate in all directions with equal speed, the wave front will have a spherical shape, this wave is called *spherical* (see Figure 3.10).

If the wave front is a plane this wave is called a *plane*.

Wave equation is the bias dependence of the oscillating point from the coordinates and time:

$$\psi = f(x, y, z, t)$$

Consider a plane wave propagating along the x axis. Each point of the medium performs harmonic oscillations, which can be described by the law:

$$\psi = A\cos(\omega t + \varphi)$$

Up to a point with an arbitrary coordinate x indignation comes after a time τ equal to $\tau = x/u$, where u is the speed of propagation of the wave.

Consequently, oscillations at this point will be delayed. The equation of a traveling wave will be different from the wave equation, and a plane wave will look like:

$$\psi = A\cos\left[\omega\left(t - \frac{x}{u}\right) + \varphi\right] \tag{3.23}$$

The value ψ represents the displacement of any medium points with the coordinate x in some fixed time t (assuming that is not the energy losses in the medium, then the amplitude is identical for all points). The expression $\left[\omega\left(t - \frac{x}{u}\right) + \varphi\right]$ is called *the wave phase*. Since the function ψ is a periodic function, the wave phase is repeated through 2π. Graph of depending, presented of equation (3.23), is like instantaneous photo wave oscillating displacement of points in given moment of time (see Figure 3.11).

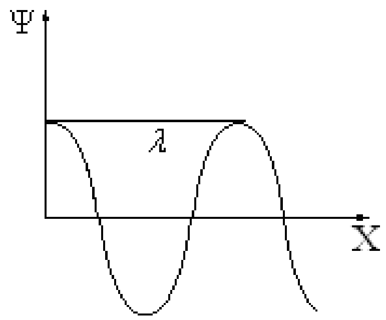

Figure 3.11

The distance between the two nearest points, oscillating in similar phases, is called the wavelength and indicated as λ. The period of wave T is the time of one complete oscillation of its points.

The wave velocity u is determined by the speed of propagation of oscillations from one medium point to another. The propagation velocity is related to the wavelength λ and wave period T as follows: $u = \frac{\lambda}{T}$.

3.9. Sound

If the elastic waves propagating in the air have a frequency in the range of about 16 to 20,000 Hz, they cause a sensation of sound reaching of the human ear.

When considering sound, there are three main aspects. Firstly, there must be a sound source; and, as for any other wave, the acoustic wave source is the body vibration. Second, energy is transferred from the sound source in the form of longitudinal sound waves, and, thirdly, the sound is registered (perceived) by our ear or device.

It is usually assumed that the sound propagates in the air, because as a rule, the air is contacted with our eardrums, and these oscillations cause the membrane to make oscillations. However, sound waves can propagate in other substances. The swimmer can hear beats of two stones against each other while he is under water, as the vibrations are transmitted to the ear with water. If you put your ear to the ground, you can hear the train or tractor coming. In this case, the earth does not act directly on your eardrums. However, the longitudinal wave propagating in the ground called a sound wave, because its oscillations lead to vibrations of air in the outer ear. Indeed, the longitudinal waves propagating in any material medium, often is called sound frequency. Obviously, the sound may not be extended in the absence of the substance. For example, one cannot hear the bell, inside the receptacle, which is evacuated of air.

The speed of sound in different materials has different values. In the air, which temperature is 0°C and 1 atm. pressure, sound travels at a speed of 331.3 m/s. The speed of sound depends on the modulus of elasticity and density of substance. In liquids and solids which are significantly less compressible and therefore have high elastic moduli, the sound speed is higher, respectively, than in gases.

Oscillations frequencies of the sound waves are in the range from 16 Hz to 20,000 Hz. If the oscillation frequency below 16 Hz, these waves are called *infrasound*, above 20000 Hz – *ultrasound*.

3.10. Features of infrasound and ultrasound

Experience shows that infrasound waves are weakly damped. Therefore the weakening of infrasound waves causes only a redistribution of the energy on the rising wave front if the wave is close to spherical. If the source is the wind on waves of the sea, where the wave front length is hundreds of meters, their infrasonic wave intensity varies less with distance. Apparently, fish and marine animals have sensitivity to infrasound, so they feel the approach of storms. Powerful infrasound waves generated during a storm, practically free of damping extend into the sea to a distance of hundreds and thousands of kilometers and signal its approach.

Ultrasonic waves are distinguished from ordinary audible sound by higher frequency oscillations, so the length of the ultrasonic wave is much less than the wavelength of sound. Due to the small wavelength, the ultrasonic wave is not diffracted. Therefore, they can be derived in the form of directional beams, similar to light beams. Reflection and refraction of ultrasonic waves occurs according to the laws similar to the laws of reflection and refraction of light.

The intensity of the ultrasonic wave is proportional to the square of wave amplitude and the square of frequency oscillations; therefore ultrasonic waves have a higher intensity. High frequency allows the reception of waves with intensities up to 100 W/cm^2 = 10 kW/m^2. At such high intensities the ultrasonic wave affects the properties of the substance and the progress of technological processes. If high intensity ultrasound is distributed in the liquid, the large tension in decompression points can lead to the formation of voids, i.e., to rupture fluid. This phenomenon is called *cavitation*. The development of cavitation is facilitated by gas bubbles, which are always present in liquids. The tremendous pressure that develops when shuts cavitation bubble can be used for crushing and grinding of various substances. After sonication, the size of the solid particles of the body which is in fluid decreased significantly, and the mixture (a suspension) becomes more homogeneous.

Placing the ultrasonic transducer into a vessel which contains two immiscible liquids (e.g. water and oil); after some time we get a homogeneous emulsion with particle sizes ranging from fractions of a micrometer to several micrometers. This effect can be used to produce new types of drugs by creating an aqueous emulsion of insoluble drugs.

The action of ultrasound significantly accelerates the processes related to the passage of fluid in porous media, which in turn speeds up many chemical and technological processes.

Also in medicine ultrasound can be used for welding of broken bones, diagnostic investigations etc. The biological action of ultrasound (which leads to microbial death) can be used for the sterilization of drugs and medical instruments.

CONTROL QUESTIONS

1. Free harmonic oscillations in an ideal system.
2. Damped oscillations.
3. Forced oscillations.
4. Resonance.
5. Mechanical oscillations.
6. The equation of wave.
7. The energy of the waves.
8. Sound.
9. Physical and physiological characteristics of sound.

THE TEST TASKS

Select one or more correct answers

1. FORCED OSCILLATIONS OCCUR UNDER THE INFLUENCE OF
   1) the elastic force
   2) conservative forces
   3) a restoring force
   4) the periodic force

2. RESONANCE OCCURS
   1) if the vibrations are not damped
   2) if the natural frequency coincides with the frequency of the driving force
   3) if the same periods of vibration acting force and the oscillating body
   4) when a large number of repetitions of vibrations

3. OSCILLATIONS of MEDIUM PARTICLES IN THE LONGITUDINAL WAVE ARE PERFORMED
   1) in all directions
   2) in the direction of wave propagation
   3) perpendicular to the direction of wave propagation
   4) perpendicular to the beam path

4. PERIOD OF THE WAVE IS …

1) the wave of oscillation points
2) during one complete oscillation wavelength points
3) the time of wave propagation in space
4) the propagation time of the waves between the two closest points

CHAPTER 4. LIQUIDS

A common property for liquids and gases is that these environments have identical properties in different directions. As a result, the external pressure exerted on a liquid or gas is transmitted equally in all directions (Pascal's law).

There are two main types of flows of liquids and gases. If the flow is smooth and contiguous layers as it slides relative to each other, it is called laminar or layered. A characteristic feature of the laminar flow is that each particle of liquid (gas) moving along a smooth trajectory and the trajectory of different particles do not overlap (see Figure 4.1a). When the flow rate exceeds a certain limit, which depends on a number of factors, the flow becomes turbulent. Turbulent flow is characterized by indiscriminate small "whirlpools", which are called vortices (see Figure 4.1b).

Vortices absorb huge amounts of energy, and although the friction called viscosity exists in a laminar flow, the viscosity is considerably higher in turbulent flow. Laminar flow can be easily distinguished from turbulent. If you drop in a moving liquid a little ink or food coloring, it instantly will stain all of the liquid in turbulent flow. As for laminar and turbulent flow can highlight the most important features:

1) liquid (gas) can be viewed as either compressible or incompressible as the environment;

2) the viscosity or internal friction, occurs in any flow of liquid (gas), but the viscosity can often be neglected. In the beginning we will consider the inviscid (ideal) flow, and then we study the effect of viscosity;

3) the flow can be established (stationary). The rate of that flow at any space point does not change over time (this does not mean that it cannot be different at different points in space). If the velocity at a given point varies with time, then this flow is called non-stationary. We are interested mainly in stationary flows;

4) the flow may have vortices or be vortex-free. In the vortex-free flow total momentum with respect to any point is equal to zero. In other words, if we entered anywhere into the flow a tiny turntable with blades, then it would not rotate. If turntable has twirled as in a funnel, or whirlpool, the flow would be a vortex (turbulent).

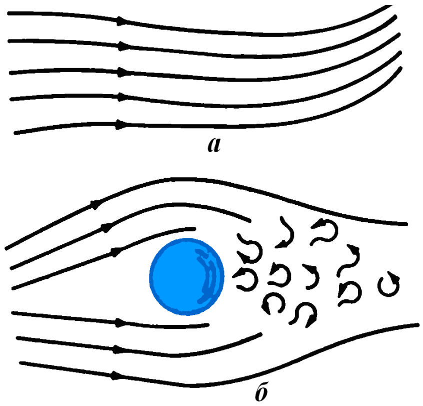

Figure 4.1: a – laminar flow; b – turbulent flow

4.1. Current lines and current tubes

In the steady liquid laminar flow (or gas), the trajectory along which a given particle travels is called the current line (see Figure 4.1a).

Fluid speed at any point is tangential to the current line (see Figure 4.2). The current lines do not intersect with each other, since otherwise at their point of intersection the speed would have been mixed. Part of the fluid (or gas) bounded by the current lines, as shown in Figure 4.3, is called the current tube.

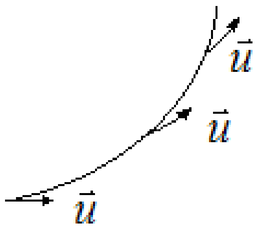

Figure 4.2

Consider the steady flow of liquid. Velocity vector at each point in space will remain constant over time. The velocity vector, each point being tangential to the current line, is tangent to the surface of the current tube; therefore, the fluid particles during its movement do not intersect the current tube walls. If during the time Δt the volume of liquid V entered into the current tube, the same liquid volume in the same time to be released from the current tube

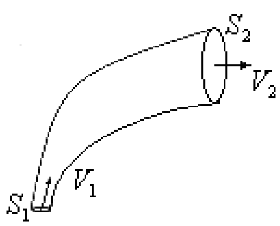

Figure 4.3

Relocating in volumes of liquid such a space of time Δt:

$$V_1 = S_1 u_1 \Delta t,$$

$$V_2 = S_2 u_2 \Delta t,$$

where u_1 and u_2 are flow velocities in sections S_1 and S_2 respectively, because the volume of the same $V_1 = V_2$, then $S_1 u_1 = S_2 u_2$. This equation, derived for the two sections of the flow of an incompressible fluid, is called the *equation of continuity*.

In the general case, for an ideal liquid under stationary conditions, the product of the velocity on the cross section of the current tube remains unchanged at any cross section of the tubes, i.e. continuity equation is:

$$S u = \text{const}$$

From the continuity equation it follows that at the narrower sections of the current tube fluid flow speed must be greater than at wider sections. We know that the capillary bed in the circulatory system has a sufficiently large cross-sectional area, so the capillary bed blood current speed will be significantly lower than in the arteries.

4.2. Bernoulli equation. The pressure in the fluid flow

The stationary flow ideal fluid cut out of the current fine tube sections S_1 and S_2. In the input section S_1, pressure p_1, velocity u_1 and height of the section over an arbitrary level h_1; in the output section S_2 respectively p_2, u_2, h_2 (see Figure 4.4).

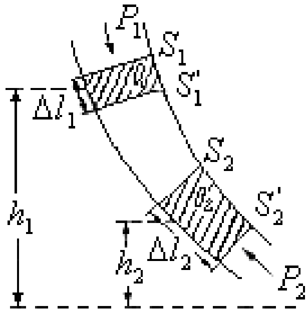

Figure 4.4

During the period of time Δt the mass included in the selected portion of the thin current tube is equal to the mass of the fluid liquid coming from this portion of the tube.

Mass of liquid m, flowing in time Δt through the section S_1, has a kinetic energy equal to $m u_1^2/2$, and potential energy mgh_1.

As a result of pressure forces in the section S_1 and S_2 side fluid layers to the left of S_1 the right side of S_2, performed work:

$$A = p_1 S_1 \Delta l_1 - p_2 S_2 \Delta l_2,$$

where Δl_1 – a path that passes the liquid column of mass m in a time Δt, a path $\Delta l_1 = u_1 \Delta t$ in section S_1, and in section S_2 let $\Delta l_2 = u_2 \Delta t$. Therefore, the work A, done by the fluid flow, is:

$$A = p_1 S_1 u_1 \Delta t - p_2 S_2 u_2 \Delta t$$

The total energy of the fluid flow flowing in time Δt through the inlet section S_1, will be equal to:

$$W_1 = \frac{m u_1^2}{2} + mgh_1,$$

and through the section S_2:

$$W_2 = \frac{m u_2^2}{2} + mgh_2$$

Changing the full energy of the fluid is equal to the work done by external forces, i.e.,

$$\frac{m u_2^2}{2} + mgh_2 - \frac{m u_1^2}{2} - mgh_1 = p_1 S_1 u_1 \Delta t - p_2 S_2 u_2 \Delta t \qquad (4.1)$$

According continuity equation flow amounts included in S_1 for the time Δt, and go through S_2 are the same, so we can write:

$$S_1 u_1 \Delta t = S_2 u_2 \Delta t = V$$

Expanding the left and right side of the equation (4.1) by the volume V and density using the formula $\rho = m/V$, the two different sections of the current tube:

$$\frac{\rho u_1^2}{2} + \rho g h_1 + p_1 = \frac{\rho u_2^2}{2} + \rho g h_2 + p_2,$$

i.e. for each section of the tube current expression is valid:

$$\frac{\rho u^2}{2} + \rho g h + p = const, \qquad (4.2)$$

Expression (4.2) is called *the Bernoulli equation*. The term p is called static pressure, ρgh – hydrostatic pressure, and $\frac{\rho u^2}{2}$ – dynamic pressure.

As a consequence of the Bernoulli equation we consider two cases: the horizontal fluid flow and expiration of the fluid from an aperture.

1. When the horizontal portion of the fluid flow stream tube all lie on the same level, then *h=const*, therefore, in equation (4.2) *ρgh* expression can be transferred to the right side and combined with a constant and then:

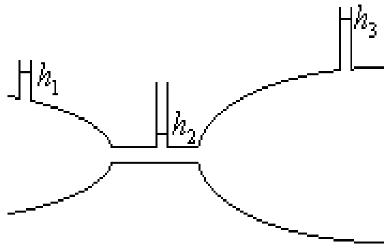

Figure 4.5

$$\rho\frac{u^2}{2}+p=const,$$

i.e. horizontal motion of the liquid amount of dynamic and static pressure in the absence of friction does not change or static pressure non-viscous fluid is increased where the velocity of its flow is reduced and vice versa. Consider a horizontally disposed tube having a variable cross section (see Figure 4.5). From the continuity equation, which is the greatest velocity, so the static pressure in this area is the smallest in the narrowed portion of the tube; can ensure that it will be less than atmospheric pressure, and then the jet liquid flowing through the tube may exert a suction effect (water-jet pump, inhalers).

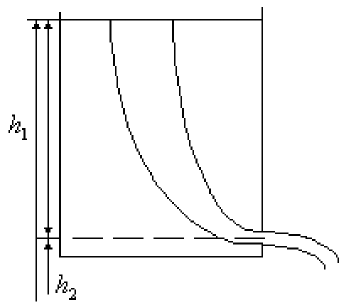

Figure 4.6

2. We apply Bernoulli's equation (4.2) to the case of the expiration of the liquid from a small aperture in a wide open container (see Figure 4.6). In this case, we assume that the external pressure (e.g., atmospheric) is constant. Let’s distinguish in liquid a tube of current having its cross section on the one side the open surface of the liquid in the vessel, and on the other side the hole through which the liquid flows.

In cross section, located on the surface of the liquid the velocity is the same throughout the cross section and equal to 0. In the section through which fluid flows, its velocity is the same everywhere and equal u. The static pressure in the other section and equal to the atmospheric pressure. With regard to this case, equation (4.2) can be written as follows:

$$\rho g h_1 = \frac{\rho u^2}{2} + \rho g h_2,$$

Where u is the velocity of the liquid from the opening of small cross-section.

Cutting the ρ and entering the $h=h_1 - h_2$ – height of the open surface of the liquid above the opening, we get:

$$\frac{u^2}{2} = gh,$$

from which:

$$u = \sqrt{2gh}, \tag{4.3}$$

expression (4.3) is called the formula Torricelli.

Thus, the velocity of liquid outflow opening located at a depth h below the open surface coincides with the velocity acquired by any body falling from the height h.

It should be remembered that this result was obtained on the assumption that the fluid is ideal. For real fluids escape velocity is less, and the lower, the greater the viscosity of the fluid. Equation (4.3) is valid for both sides and for the bottom opening does not depend on the angle of inclination of the outlet opening.

Formula Torricelli, for example, allows us to estimate the height of the vessel location with the drug when administered into a vein.

Inhaler. This device is used to introduce drugs into the cavity of the nasopharynx in a spray form (see Figure 4.6a).

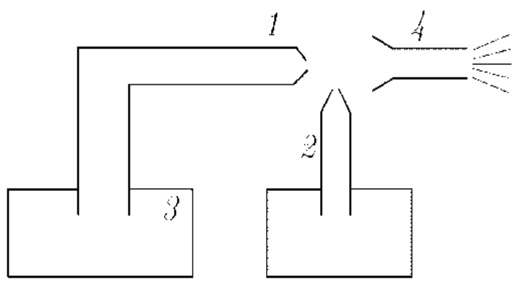

Figure 4.6a. Driving inhaler

The inhaler consists of two tubes disposed at a right angle. Horizontally located tube has a narrowing at the end (1). Slightly below the upper end is the end vertical tube (2) whose lower end is lowered into the vessel with a liquid formulation (3). In a horizontal tube supplied steam. When passing the tapered end of the steam velocity increases and pressure drops. In the area of reduced pressure, a drug is sucked in, which

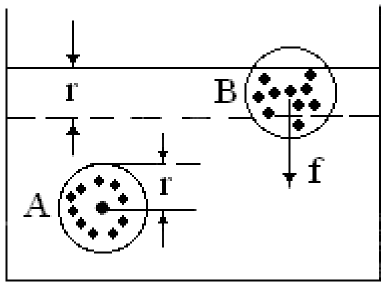

Figure 4.7

is sprayed with a stream of steam. The result is a mixture of steam, air and the droplets of the drug, which through the pipe (4) is applied to the patient.

4.3. Surface tension

The forces of molecular interaction rapidly decrease with distance. Therefore, we can assume that each molecule interacts only with those that lie within the sphere of radius r – the sphere of molecular action with the center in given molecule (see Figure 4.7).

If some molecule A located deep inside the liquid, the forces acting on it from other molecules are mutually compensated. The situation is different with the molecules located on the surface of the liquid. The concentration of the molecules in the vapor above the liquid is much less than in the liquid. Therefore, some attractive force f, directed inside the liquid, acts on the molecule B that lies in the upper layer of the liquid.

Thus, for each molecule, which lies at a distance of less than r, from the surface of the liquid, from the other molecules it has been acting force, which is directed inside of the liquid.

Hence, the entire surface layer is situated in a specific condition: it exerts pressure on the fluid like the pressure of an elastic film. In accordance with this the molecules of surface layer have additional potential energy.

Moving the molecule from the surface layer deeper into the liquid is accompanied by the performance of work, and the potential energy which it had possessed: a molecule in the surface layer is converted into kinetic energy of the molecule.

Conversely, the transition of molecule from the liquid depth in the surface layer requires the expenditure of work for overcoming force f (see Figure 4.7). Work is performed on the molecule by its kinetic energy. The potential energy of molecule trapped in the surface layer increases.

In the liquid under given external conditions equilibrium is established when the number of molecules in the surface layer does not vary over time. If for one reason or another surface of the liquid increases, the number of molecules moves from the depth of liquid to the surface layer. For this it is necessary to expend external work dA which is proportional to an increase in surface area dS, i.e.

$$dA = -\sigma\, dS$$

the minus sign indicates that the increasing of the surface takes place only when work is performed by external forces.

Coefficient σ which characterizes the properties of the liquid surface is called surface tension. It is measured by the work necessary to increase the liquid surface at a constant temperature per unit area.

In the SI system, the surface tension has the dimension [J / m^2].

Work expended on increasing of the liquid surface area, results in an increase the potential energy of the liquid surface, i.e. the potential energy is proportional to the area S of the surface layer. Every system tends to a state of minimum energy. In free state, the liquid is committed to ensuring that its surface was minimal (free liquid droplets always have a sphere shape).

The molecules of the surface layer are on average at larger distances from each other than the molecules within the liquid. The liquid in the surface layer is always in a stretched, stressed state, and therefore the force of surface tension of the liquid acts on the free surface boundary, which is directed concerning the liquid surface and normal to the free surface:

$$F = \sigma L,$$

where L – contour length, bounding surface of the fluid. Surface tension coefficient is determined by the force of surface tension. Then the second definition of the surface tension can be formulated as follows: the surface tension is numerically equal to the force of surface tension acting on a unit length of the contour, limiting the surface of the liquid.

With increasing temperature, the interaction of liquid molecules is somewhat weakened, as by increasing the molecules kinetic energy, the liquid "loosens" and the average distance between the molecules increases. Therefore, with increasing temperature the value of the surface tension σ should decrease. In the same direction should act vapor density increase with the temperature increasing.

4.4. Wetting and non-wetting

When considering the molecular pattern of the surface layer of liquid, we note that the liquid molecules at the surface, separating the liquid and gas (air or steam of the liquid), are not attracted by gas molecules (the concentration of the gas molecules is too small). If the medium with which the liquid under consideration borders is sufficiently dense, then it is impossible to neglect the interaction of molecules of the medium with molecules of the liquid. Because of this, the surface tension of the liquid, which borders with different environments, can vary considerably among them. For example, at room temperature, the coefficient of surface tension of water at the water-benzene $\sigma = 0{,}033$, and at the water-air $\sigma = 0{,}012$, water at the interface with its own vapor $\sigma = 0{,}073$ N / m.

Surface tension at the boundary of different environments plays an important role in a variety of surface phenomena. Let us consider surface phenomena arising at liquid contact with a solid. If the liquid is in the vessel, then, in addition to the free surface, there is also an interface between the liquid and the solid (the walls of the vessel). The molecules of the liquid are in contact with the vessel wall, interact with their immediate neighbors, both liquid and solid. If the force of liquid molecules interacts with a molecule of liquid more force liquid molecules interact with the solid-state molecule, the resulting force is directed deep into the liquid (see Figure 4.8).

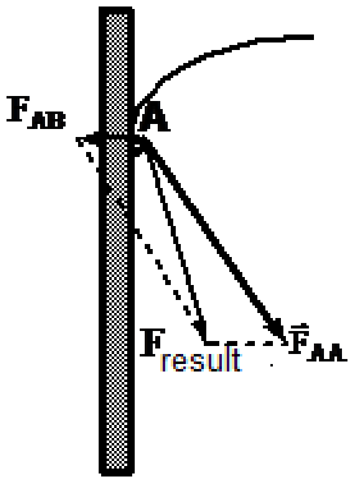

Figure 4.8

That liquid is called non-wetting liquid. The free surface of the liquid at the edges of the vessel wall is convex. If the force of liquid molecule interaction with a liquid molecule is less than the liquid molecule interaction with a solid molecule, the resultant force acting on a molecule of liquid held by a solid body wall, directed into the solid body, i.e. liquid molecule as if sticks to the solid. Such liquids are called wetting (see Figure 4.9). The free surface of the liquid at the edges of the vessel wall is concave.

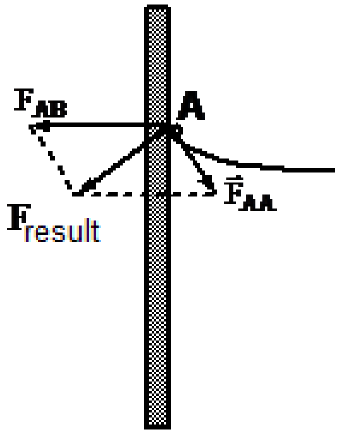

Figure 4.9

A quantitative assessment of wetting is the wetting angle. *Wetting angle* is the angle Q, composed of the vessel wall and the tangent to the surface of the liquid drawn from the liquid surface intersection point with the vessel wall. The angle is measured always inside the liquid (see Figure 4.10). On wetting of the solid body by liquid wetting angle lies in the range: $0 \le Q \le \frac{\pi}{2}$. Complete wetting happens when wetting angle is zero $Q = 0$.

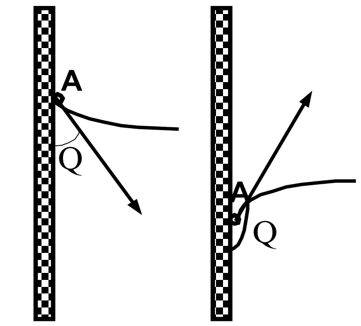

Figure 4.10

If the liquid is non-wetting, the wetting angle is in the range $\frac{\pi}{2} \le Q \le \pi$. Complete non-wetting happens in case when the wetting angle is equal to π.

4.5. The dependence of the molecular pressure on the curvature of the liquid surface

The interaction of the liquid particles with solid particles affects the shape of the surface of the liquid poured into vessel. In the walls of the vessel surface of the liquid is curved (see Figures 4.8, 4.9). The narrow tubes (capillaries) or a narrow gap between the two walls the whole surface of the liquid is curved. If the liquid wets the walls then surface has a concave shape (see Figure 4.11a), if it does not wet the walls then surface has a convex form (see Figure 4.11b). The curved liquid surfaces in the vessels are called meniscuses. Due to the action of surface tension forces within the fluid, pressure will be different by a certain amount Δp from the external gas or vapor pressure above the liquid surface.

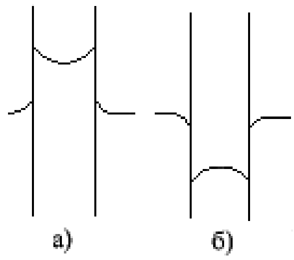

Figure 4.11

Estimate the value of additional pressure in the case of the spherical surface layer. We distinguish in the surface of a sphere smaller spherical segment. Surface tension forces applied to the contour of the segment are directed at a tangent to the spherical surface (see Figure 4.12). To contour element Δl, shown by fatty curve is applied the force Δf, equal in absolute value $\Delta f = \sigma \Delta l$.

Let us find component Δf_1 of this force parallel to the radius of curvature of the OC. From the figure we have $\Delta f_i = \Delta f \sin\varphi = \sigma \Delta l \sin\varphi$.

It is precisely this component that creates additional pressure. We found a component of the surface tension force acting on the contour element Δl. The total force applied to the contour and creating additional pressure is obviously equal to the sum of all forces acting on the individual contour elements.

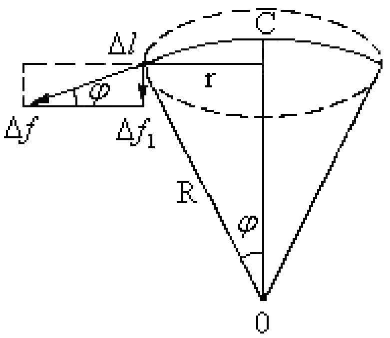

Figure 4.12

$$\Delta f_1 = \sum \Delta f_i = \sigma \sin\varphi 2\pi r$$

From the figure 4.12 follows that $\sin\varphi = r/R$, so:

$$f_1 = \sigma 2\pi r^2 / R$$

The additional pressure Δp is obtained by dividing the force f_1 on the segment area πr^2:

$$\Delta p = f_1 / \pi r^2 = 2\sigma / R \qquad (4.4)$$

The ratio (4.4) gives the value of the additional pressure under the spherical surface and is known *as Laplace's formula*. In the case of the concave surface the resulting force of the liquid surface tension directed from liquid to gas (vapor). In this case $\Delta p = -2\sigma / R$, e.g. pressure inside fluid under a curved surface is less than that of the gas above the liquid surface by an amount Δp.

4.6. Capillary phenomena

In narrow tubes (capillaries) due to wetting or non-wetting capillary wall by liquid the curvature of the liquid surface (i.e., meniscus) becomes significant. This occurs when additional pressure Δp causes an appreciable lifting or lowering of the liquid level.

For example, consider the case of round the capillary radius r, immersed in a large vessel with the liquid which does not wet the capillary wall. Thus meniscus within the capillary is formed, and under the influence of additional pressure Δp a liquid in the capillary is lowered to a certain depth, as shown in Figure 4.13.In broad vessel due to the action of gravity force can be regarded as substantially planar liquid surface. In the narrow tube, on the contrary, it is possible to neglect the influence of gravity forces compared to the forces of surface tension and the liquid surface can be regarded as sphere of radius R. The Figure 4.13 shows that the

$$R = \frac{r}{\cos\theta},$$

where θ – wetting angle on the border "liquid – solid wall".

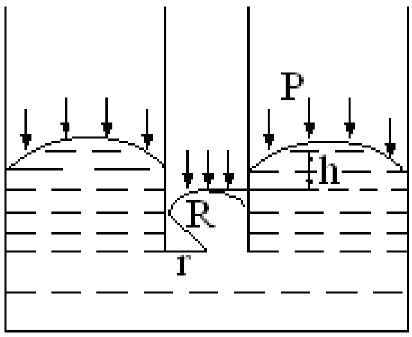

Figure 4.13

At the level of the liquid surface in the capillary pressure in the liquid is equal:

$$p + \Delta p = p + \frac{2\sigma}{R},$$

where p – external gas pressure. According to the law of communicating vessels, it must be equal to the total pressure at the same level in a wide vessel

$$p + \rho g h,$$

where ρgh – the hydrostatic pressure of the liquid column density ρ at depth h. Equating obtains:

$$p+\frac{2\sigma}{R}=p+\rho gh,$$

From whence

$$h=\frac{2\sigma}{\rho gR}=\frac{2\sigma\cos\theta}{\rho gr}$$

Exactly the same expression is obtained for the height of a liquid lifting wetting the walls of the capillary with radius r. When fully wetted (e.g., water-glass) $\theta=0$, the radius of the meniscus is equal to the radius o of capillary and the height of the liquid lifting is equal to

$$h=\frac{2\sigma}{\rho gr}$$

Capillary phenomena play an important role in medicine. In order for a liquid to be not only drawn into the capillary, but generally to penetrate into the pores requires a small wetting angle. When we have a large value of the wetting angle the objects will remain dry. Here are some examples that demonstrate the role of capillarity and wetting.

1. Systems which need a small wetting angle (preferably with a large surface tension): blood on the bandage (hygroscopic cotton wool), drops from the rhinitis on the nasal mucosa, saliva on the food. The solvent for a paint on the dry powder colorant, liquid paint on painted surfaces (different colors of tablets), etc.

2. Systems which need a large wetting angle: water on the glasses (small droplets evaporate quickly), protective cream, etc.

4.7. The surfactants

Many substances reduce the surface tension of liquids. Those substances called *surfactants* (soap, oleic acid).

To achieve a minimum of surface energy the surfactant should be concentrated near a surface of liquid or solid. Increasing the concentration of dissolved substances which reduce the surface tension near a surface of the liquid or solid, it is called *adsorption*. It can occur not only on the free liquid surface, but also on the contact surface between two liquids or on the surface of a liquid and a solid. For adsorption it is only necessary that the solute lowers the surface tension on that surface. Bodies on the surface which concentrates the surfactants are known as adsorbents. The simplest surfactants relative to the water are alcohols, fatty acids and their salts.

4.8. Transfer phenomena

If liquid or gas molecules differ from one another by a characteristic quantity (mass, momentum, energy, etc.), and the distribution of molecules over the value of the specified characteristic is uniform, then due to thermal motion this magnitude "is transferring" from one place to another. The result is a stream of the quantity (mass, momentum, energy) conditional on a number of phenomena called transference phenomena.

Viscosity (internal friction). Let us consider flow of liquid or gas, wherein the liquid flow velocity u is the same at all points in the direction, but changing in magnitude along the perpendicular to the velocity υ. We will choose the direction of this perpendicular along the x axis (see Figure 4.14.); then the velocity is a function of the coordinates x: $u = u(x)$. We can say that the flow is divided into layers parallel to each other, moving at a different velocity, but parallel to each other.

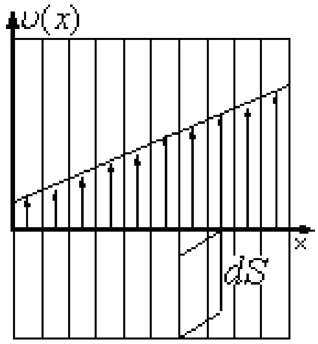

Figure 4.14

Due to the thermal motion the molecules move from one layer to another, carrying with them the impulse ($m\upsilon$) of its direction of motion. The result is a momentum transfer process of the layers where the flow rate is greater in those layers where it is less and vice versa. Due to the transfer of momentum from fast to slow layers and vice versa there is a change of momentum layers (fast layers is slowing, slow - accelerating). Newton's Second Law asserts that the momentum change can only take place under the force of momentum: $\vec{F}\Delta t = \Delta m\vec{u} = \Delta\vec{K}$. So, between the layers there are forces that are called the forces of internal friction. A process that leads to the velocities equalization of the flowing of different layers called *internal friction or viscosity*.

The basic equation, which describes the force of internal friction, as follows:

$$F = -\eta \frac{du}{dx} dS$$

and is known as the *Newton equation*. The equation includes a coefficient η called *viscosity coefficient*. From Newton's equation it follows that viscosity coefficient is:

$$\eta = \frac{F}{\frac{du}{dx} dS},$$

i.e. viscosity coefficient is a physical quantity that is numerically equal to the force acting between the layers of liquid, which contact area is equal to 1, and the velocities gradient is also equal to 1. The value $\frac{du}{dx}$ is called the *velocity gradient*. The velocities gradient indicates a change in the liquid flow velocity in the direction perpendicular to the flow of a liquid.

The liquid viscosity is usually many times greater than the gas viscosity. With increasing temperature, liquid viscosity decreases rapidly, and gas viscosity increases slowly.

For many liquids, the viscosity of the liquid depends only on the temperature and pressure. These liquids are called *Newtonian*.

Non-newtonian liquids are liquids in which at constant temperature and pressure the viscosity depends on the velocity gradient and other factors. Non-newtonian liquids include blood, which is a highly dispersed suspension.

The viscosity of the liquid determines the resistance force of the liquid movement of bodies in it. Thus, the resistance force of the ball movement of radius *r* at low velocities is by Stokes equation

$$F = 6\pi r \eta u.$$

This force is directly proportional to the viscosity of the environment η and velocity of the ball movement$u$. You can find the viscosity of environment if it will measure the velocity of the ball.

Transfer of substance molecules due to thermal motion of the region where the concentration of the substance is greater to the region of lower concentration is called *diffusion*. As a result, the diffusion concentration equalization occurs. Mass of substance diffused through area ΔS in a time Δt, determined by *Fick's law*:

$$\Delta M = -D\frac{dC}{dx}\Delta S\Delta t,$$

where C – the concentration of diffusing molecules, $\frac{dC}{dx}$ – gradient of this concentration, D – diffusion coefficient, which depends on the properties of the diffusing material and the conditions in which it resides.

Note that the higher temperature of the liquid, the smaller number of oscillations in the molecule makes its equilibrium position to hopping, the more intense the diffusion process and the higher value of D. However, diffusion coefficients in liquid still have order 10^{-5}sm^2/sec, which is much smaller than in gases.

Thermal conductivity is a phenomenon that molecules from a hot layer with temperature T_1, where they have a high kinetic energy, penetrate into the layer with the colder temperature T_2 and transferred its kinetic energy, which creates a heat flow Q. The law of thermal conductivity established by Fourier:

$$\Delta Q = -\chi\frac{dT}{dx}\Delta S\Delta t,$$

where χ – thermal conductivity coefficient, $\frac{dT}{dx}$ – temperature gradient. Coefficient of thermal conductivity does not depend on the pressure, but depends on the temperature, can be approximately assumed that $\chi \approx \sqrt{T}$.

4.9. Laminar and turbulent flow of fluid

We have already discussed that there are two flows of a liquid or gas: laminar and turbulent fluid flow. Typically, the laminar flow of the fluid is set in the tubes with smooth walls without sharp changes of cross-sectional areas or sudden bends of the tube. In case of violation of these conditions and at high velocities of fluid flow, fluid flow becomes turbulent, i.e. vortex (see Figure 4.1).At the vortex flow the fluid flow velocity is different at different points of the spiral, then local changes of pressure in the fluid will be observed, causing the oscillatory movement of particles, accompanied by sound effects (noise, murmur), due to the turbulent flow can be easily detected.

English scientist Reynolds found that the nature of the flow depends on the value of the dimensionless quantity:

$$\mathrm{Re} = \frac{\rho u D}{\eta}, \tag{4.5}$$

where ρ is the liquid or gas density, u is the average flow velocity over the cross section of the tube, η is the fluid viscosity constant, D is typical for cross-sectional size, for example, a square side with a square cross-section, the radius at the round cross-section.

The value of Re is called the *Reynolds number*. At low values of Reynolds number laminar fluid flow has been observed. Starting with a certain value Re, called critical, flow gains a turbulent nature. If the diameter is taken as the characteristic dimension for a round tube, then the critical Reynolds number is approximately 2000 units. The nature of flow of various gases or liquids in the tubes of different cross sections will be absolutely identical, if each flow correspondence same value of Reynolds number.

4.10. Poiseuille formula

The volume of fluid flowing through a cross section of a circular tube per unit time in laminar flow depends on the viscosity of the fluid, pressure difference and the size of the tube through which the fluid flows. Denote dQ - volume velocity of the fluid, this velocity is the volume of fluid flowing through the cross section of the tube per unit time:

$$dQ = \frac{dV}{dt} \tag{4.6}$$

Let us find the connection between the linear and volumetric of fluid velocities. Allocate in the tube small volume in the form of a cylinder, the cross sectional area of this cylinder is equal to dS, and length equal to dl, then the volume of the cylinder $dV = dS\,dl$. The length of the cylinder can be expressed through a linear flow velocity of fluid as follows: $dl = udt$, then $dV = dS\,udt$. Substituting this expression in equation (4.6) we obtain:

$$dQ = \frac{dV}{dt} = \frac{dSudt}{dt} = dSu \qquad (4.7)$$

Thus, the volume velocity of fluid flow through the vessel is equal to the linear velocity of fluid flow, multiplied by the cross-sectional area.

Flow of viscous fluid through the tubes presents a particular interest for medicine, as a circulatory system consists essentially of cylindrical containers with different diameters. French physician, Jean Louis Marie Poiseuille, was working on circulation and respiration, and interested in the problems of hydrodynamics. He obtained a formula, which was called the Poiseuille formula. Formula (4.7) establishes a relationship between volume velocity of fluid flow, viscosity of fluid and pressure difference at the ends of the cylindrical tube in laminar flow:

$$Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l}, \qquad (4.8)$$

where R – the inner radius of the tube at which the liquid flows, $p_1 - p_2$ – pressure difference at the ends of the tube, η – viscosity of fluid flowing through the tube.

If the liquid did not have a viscosity, then for the flow through the horizontal pipe it would not be necessary to exert any force. But due to viscosity, the stationary flow of any real liquid in the tube is possible only when a pressure difference is created between the ends of the tube.

According to the Poiseuille formula flow of fluid Q is proportional to the pressure gradient $(P_1 - P_2)/l$ and inversely proportional to the viscosity of the fluid, which was expected. However, it may seem surprising that Q depends on *fourth* degree of the tube radius. This means that for the same pressure gradient increasing of the tube radius in twofold will increase the flow of fluid by 16 times! Thus, even a small changing in the tube radius results in a significant change in the fluid flow; so as to maintain the flow at the same level, it would have to significantly change the pressure difference.

An interesting example of the relationship kinds of R^4 may be found in the circulatory system of the human body. However, as the Poiseuille formula is only valid for the laminar flow of an incompressible fluid with constant viscosity η, it may not exactly be carried out for blood. The fact that it is not completely laminar flow of blood, blood contains suspended particles, whose diameter nearly equals the diameter of the capillary, and its viscosity η depends on the flow velocity. However, in this case Poiseuille formula is a good approximation of the first order. The flow of blood in the body is regulated by tiny muscles surrounding the blood vessels. With the reduction of these muscles is reduced vessel diameter and the flow which, in accordance with the formula Poiseuille proportional R^4, decreases sharply even at a small radius decreases. Thus, the hardly noticeable contractions of these muscles are very precisely controlled blood supply to various organs. However, if, say, due to arteriosclerosis (solidification of the vessel walls) and vascular deposits of cholesterol the radius decreases, then requiring a higher pressure gradient to maintain of normal blood flow. If the vessel radius is halved, then the heart will have to increase the pressure by 16 times. Under these conditions the heart is overloaded, but usually cannot provide the desired value of fluid flow, i.e. normal blood circulation.

Thus, high blood pressure indicates that the heart is overloaded and that the flow of blood through the arteries is below the normal value.

CONTROL QUESTIONS

1. Hydrodynamics.
2. The equation of continuity of the jet.
3. Bernoulli equation.
4. The suction effect of the jet.
5. Measurement of statistical and dynamic pressure.
6. The viscous liquid.
7. Laminar and turbulent flow.

THE TEST TASKS

Select one or more correct answers.

1. THE EQUATION HAS THE FORM OF CONTINUOUS JET

1) $Su = \text{const}$
2) $\frac{S}{u} = 0$
3) $Su^2 = \text{const}$
4) $S^2u = \text{const}$

2. TUBULAR SURFACE FORMED LINES CURRENT WITH AN INFINITELY SMALL CROSS SECTION, CALLED

1) flow tube
2) the flow tube
3) the current line
4) elementary stream

3. BERNOULLI EQUATION FOR TWO DIFFERENT SECTIONAL FLOW MAKES THE RELATIONSHIP BETWEEN

1) pressure, flow and liquid flow rate
2) rate, pressure and viscosity coefficient of fluid flow
3) pressure, velocity and liquid flow geometric height
4) geometric altitude, velocity, fluid flow rate

4. INHALER – THIS IS A DEVICE (FOR)

1) local therapeutic effects of electric or magnetic field of ultrahigh frequency
2) for medical treatment by the action of low-frequency ultrasound energy to biological tissue affected by a liquid medication and a contact
3) for insertion into the nasopharynx region medicaments in a spray form
4) measurement of color in a color scale for comparing the intensity or color of the solution from the test standard

5. THE SURFACE TENSION CHARACTERIZING THE PROPERTIES OF

1) metal surface
2) internal friction in liquids
3) internal friction in gases
4) The liquid surface

ANSWERS TO TEST TASKS

CHAPTER 1. KINEMATICS

1. – 3)	3. – 4)	5. – 2)
2. – 4)	4. – 1)	6. – 3)

CHAPTER 2. DYNAMICS

1. – 2)	3. – 2)	5. – 3)
2. – 3)	4. – 1)	6. – 1)

CHAPTER 3. MECHANICAL OSCILLATIONS AND WAVES

1. – 4)	3. – 3)
2. – 2)	4. – 2)

CHAPTER 4. LIQUIDS

1. – 1)	3. – 3)	5. – 4)
2. – 1)	4. – 3)	

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